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کوانتش محیط‌های پیوسته در فضای فاز گسترش یافته

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Quantization of Continuous Media in Extended Phase Space

S. Nasiri*, M. T. Fallahi and H. Pahlavani****

***Department of Physics, University of Zanjan**

****Department of Physics, University of Isfahan**

Abstract

In this article, the quantum mechanical formulation in extended phase space has been generalized to the quantized many body systems with degrees of freedom limited to the evolution of classical and quantum mechanical fields. This makes the investigation of such fields possible in phase space. As an example, the problem of vibrating string has been solved using this formalism.

Keywords: Quantized media, Generalized phase space, Quantum and classic fields

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$$L(q, p, \dot{q}, \dot{p}) = -\dot{q}_1 p_1 - \dot{p}_1 q_1 + L^q(q, \dot{q}) + L^p(p, \dot{p}),$$

$L^p \quad L^q$

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_1} - \frac{\partial L}{\partial q_1} = \frac{d}{dt} \frac{\partial L^q}{\partial \dot{q}_1} - \frac{\partial L^q}{\partial q_1} = 0$$

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{p}_1} - \frac{\partial L}{\partial p_1} = \frac{d}{dt} \frac{\partial L^p}{\partial \dot{p}_1} - \frac{\partial L^p}{\partial p_1} = 0$$

$p_1 \quad q_1$

$$(p_1 = \frac{\partial L^q}{\partial \dot{q}_1}, q_1 = \frac{\partial L^p}{\partial \dot{p}_1})$$

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$$. (\quad) \quad p \quad q$$

$$p \quad q$$

$$\pi_{q_1} = \frac{\partial L}{\partial \dot{q}_1} = \frac{\partial L^q}{\partial \dot{q}_1} - p_1$$

$$\pi_{p_1} = \frac{\partial L}{\partial \dot{p}_1} = \frac{\partial L^p}{\partial \dot{p}_1} - q_1$$

:

$$H(q, p, \pi_q, \pi_p) = \pi_{q_1} \dot{q}_1 + \pi_{p_1} \dot{p}_1 - L(q, p, \dot{q}, \dot{p}).$$

$$= h(p + \pi_q, q) - h(p, q + \pi_p)$$

 $h(p, q)$

$$h\left(\frac{\partial L^q}{\partial \dot{q}}, q\right) = \frac{\partial L^q}{\partial \dot{q}_1} \dot{q}_1 - L^q(q, \dot{q})$$

$$h\left(p, \frac{\partial L^p}{\partial \dot{p}_1}\right) = \frac{\partial L^p}{\partial \dot{p}_1} p_1 - L^p(p, \dot{p})$$

:

$$\dot{q}_1 = \frac{\partial H}{\partial \pi_{q_1}}, \dot{p}_1 = \frac{\partial H}{\partial \pi_{p_1}}, \dot{\pi}_{q_1} = -\frac{\partial H}{\partial q_1}, \dot{\pi}_{p_1} = -\frac{\partial H}{\partial p_1}$$

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$$\pi_{p_1} \quad \pi_{q_1, p_1, q_1} ($$

$$[\pi_{p_1}, P_j] = [\pi_{q_1}, \pi_{q_j}] = 0$$

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$$[\pi_{q1}, q_j] = -i\hbar \delta_{1j}, \pi_{q1} = -i\hbar \frac{\partial}{\partial q_1},$$

$$[\pi_{p1}, p_j] = -i\hbar \delta_{1j}, \pi_{p1} = -i\hbar \frac{\partial}{\partial p_1}$$

$x(q, p, t)$ (

$$i\hbar \frac{\partial x}{\partial t} = Hx = \left\{ h\left(p - i\hbar \frac{\partial}{\partial q}, q\right) - h\left(p, q - i\hbar \frac{\partial}{\partial p}\right) \right\} x$$

$$= \sum_{n=1}^{\infty} \frac{(i\hbar)^n}{n!} \left\{ \frac{\partial^n h \partial^n}{\partial p^n \partial q^n} - \frac{\partial^n h \partial^n}{\partial q^n \partial p^n} \right\} x.$$

: $f(q, p)$ ()

$$\langle f \rangle = \int f(q, p) x^*(q, p, t) dq dp.$$

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$p \quad q$ (

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() $f = 1$ (

$$\int x^* dq dp = 1.$$

$$t \quad q \quad \eta(q, t)$$

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$$L^n = L^n(\eta, \frac{\partial \eta}{\partial q}, \frac{\partial \eta}{\partial t}, q, t).$$

$$t_2 \quad t_1$$

$$\delta I = \delta \int_{t_1}^{t_2} L^n dq dt = 0$$

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$$\frac{\partial}{\partial t} \left[\frac{\partial L^n}{\partial(\partial \eta / \partial t)} \right] + \frac{\partial}{\partial q} \left[\frac{\partial L^n}{\partial(\partial \eta / \partial q)} \right] - \frac{\partial L^n}{\partial \eta} = 0$$

$$t \quad p \quad \xi(p, t)$$

$$p_1 \quad q_1$$

$$\xi(p, t) \quad \eta(q, t)$$

$$L^\xi$$

$$L^\xi = L^\xi(\xi, \frac{\partial \xi}{\partial p}, \frac{\partial \xi}{\partial t}, p, t).$$

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$$\frac{\partial}{\partial t} \left[\frac{\partial L^\xi}{\partial(\partial \xi / \partial t)} \right] + \frac{\partial}{\partial p} \left[\frac{\partial L^\xi}{\partial \xi / \partial p} \right] - \frac{\partial L^\xi}{\partial \xi} = 0.$$

:

$$L = L(\eta, \frac{\partial \eta}{\partial q}, \frac{\partial \eta}{\partial t}, \xi, \frac{\partial \xi}{\partial p}, \frac{\partial \xi}{\partial t}, q, p, t).$$

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$$\delta I = \delta \int \int L dq dp dt = 0$$

$$\frac{\partial}{\partial t} \left[\frac{\partial L}{\partial(\partial \eta / \partial t)} \right] + \frac{\partial}{\partial q} \left[\frac{\partial L}{\partial(\partial \eta / \partial q)} \right] - \frac{\partial L}{\partial \eta} = 0$$

$$\frac{\partial}{\partial t} \left[\frac{\partial L}{\partial(\partial \xi / \partial t)} \right] + \frac{\partial}{\partial p} \left[\frac{\partial L}{\partial(\partial \xi / \partial p)} \right] - \frac{\partial L}{\partial \xi} = 0$$

$$\xi(p, t) \quad \eta(q, t)$$

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$$\xi = \frac{\partial L^\eta}{\partial(\partial \eta / \partial t)}$$

$$\eta = \frac{\partial L^\xi}{\partial(\partial \xi / \partial t)}$$

η

$$h \left(\eta, \frac{\partial \eta}{\partial q}, \frac{\partial L^\eta}{\partial(\partial \eta / \partial t)}, q, t \right) = (\partial \eta / \partial t) \frac{\partial L^\eta}{\partial(\partial \eta / \partial t)} - L^\eta.$$

$$h \left(\xi, \frac{\partial \xi}{\partial p}, \frac{\partial L^\xi}{\partial(\partial \xi / \partial t)}, p, t \right) = -(\partial \xi / \partial t) \frac{\partial L^\xi}{\partial(\partial \xi / \partial t)} + L^\xi.$$

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$$L\left(\eta, \frac{\partial \eta}{\partial q}, \frac{\partial \eta}{\partial t}, \xi, \frac{\partial \xi}{\partial p}, \frac{\partial \xi}{\partial t}, q, p, t\right) = -(\partial \xi / \partial t)\eta - (\partial \eta / \partial t)\xi + L' + L^\xi.$$

$$\xi \quad \eta$$

:

$$\pi_\eta = \frac{\partial L}{\partial(\partial \eta / \partial t)} = -\xi + \frac{\partial L'}{\partial(\partial \eta / \partial t)}$$

$$\pi_\xi = \frac{\partial L}{\partial(\partial \xi / \partial t)} = -\eta + \frac{\partial L^\xi}{\partial(\partial \xi / \partial t)}$$

$$\xi \quad \eta$$

$$\pi_\xi \quad \pi_\eta$$

:

$$H\left(\eta, \frac{\partial \eta}{\partial q}, \pi_\eta, \xi, \frac{\partial \xi}{\partial p}, \pi_\xi, q, p, t\right) =$$

$$\pi_\eta(\partial \eta / \partial t) + \pi_\xi(\partial \xi / \partial t) - L\left(\eta, \frac{\partial \eta}{\partial q}, \frac{\partial \eta}{\partial t}, \xi, \frac{\partial \xi}{\partial p}, \frac{\partial \xi}{\partial t}, q, p, t\right).$$

$$\frac{\partial L^\xi}{\partial(\partial \xi / \partial t)} \quad \frac{\partial L'}{\partial(\partial \eta / \partial t)}$$

:

$$H = h\left(\eta, \frac{\partial \eta}{\partial q}, \xi + \pi_\eta, q, t\right) - h\left(\eta + \pi_\xi, \xi, \frac{\partial \xi}{\partial p}, p, t\right)$$

$$\xi = \pi_\eta$$

$$\eta = \pi_\xi$$

$$H = \sum \frac{1}{n!} \left\{ \frac{\partial^n h}{\partial \eta^n} \pi_\xi^n - \frac{\partial^n h}{\partial \xi^n} \pi_\eta^n \right\},$$

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$$\xi \quad \eta \quad T \quad \mu \quad :$$

$$L^\eta = \frac{1}{2} \mu \left(\frac{\partial \eta}{\partial t} \right)^2 - \frac{1}{2} T \left(\frac{\partial \eta}{\partial q} \right)^2$$

$$L^\xi = -\frac{1}{2T} \left(\frac{\partial \xi}{\partial t} \right)^2 + \frac{1}{\mu} \left(\frac{\partial \xi}{\partial p} \right)^2$$

$$L(\eta, \frac{\partial \eta}{\partial q}, \frac{\partial \eta}{\partial t}, \xi, \frac{\partial \xi}{\partial p}, \frac{\partial \xi}{\partial t}, q, p, t) =$$

$$-\left(\frac{\partial \xi}{\partial t} \right) \eta - \left(\frac{\partial \eta}{\partial t} \right) \xi + \frac{1}{2} \mu \left(\frac{\partial \eta}{\partial t} \right)^2 - \frac{1}{2T} \left(\frac{\partial \xi}{\partial t} \right)^2 - \frac{1}{2} T \frac{\partial \eta^2}{\partial q} + \frac{1}{2\mu} \left(\frac{\partial \xi}{\partial p} \right)^2$$

ξ, η

$$\mu \frac{\partial^2 \eta}{\partial t^2} - T \frac{\partial^2 \eta}{\partial q^2} = 0$$

$$-\frac{1}{T} \frac{\partial^2 \xi}{\partial t^2} + \frac{1}{\mu} \frac{\partial^2 \xi}{\partial p^2} = 0$$

$\xi \quad \eta$

$$H = \frac{1}{2\mu} (\xi + \pi_\eta)^2 + \frac{1}{2} T \left(\frac{\partial \eta}{\partial q} \right)^2 - \frac{1}{2} T (\eta + \pi_\xi)^2 - \frac{1}{2\mu} \left(\frac{\partial \xi}{\partial p} \right)^2$$

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$$\xi \quad \eta \quad . () \quad \pi_\eta = \pi_\xi = 0$$

$$\xi \quad \eta \quad . \quad \pi_\xi \quad \pi_\eta$$

$$\xi \quad \eta$$

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X

$$\pi_\xi \quad \pi_\eta$$

X

$$\xi(p, t) \quad \eta(q, t) ($$

$$\pi_\eta = -i\hbar \frac{\partial}{\partial \eta}$$

$$\pi_\xi = -i\hbar \frac{\partial}{\partial \xi}$$

$$[\pi_\eta(q, t), \eta(q', t)] = -i\hbar \delta(q - q'),$$

$$[\pi_\xi(p, t), \xi(p', t)] = -i\hbar \delta(p - p').$$

$$[\pi(q, t), \xi(p, t)] = 0$$

$$[\pi_\eta(q, t), \pi_\xi(p, t)] = 0$$

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$$\begin{array}{ccc}
 & \pi_\xi, \pi_\eta & \xi \quad \eta \\
 \xi(p, t), \pi_\xi(p, t) & & \\
 & & \eta(q, t), \pi_\eta(q, t) \\
 x(\xi, \eta, t) \in X & & H \quad (\\
 x & & ()
 \end{array}$$

$$ih \frac{\partial x}{\partial t} = Hx = \sum \frac{(-ih)^n}{n!} \left\{ \frac{\partial^n h}{\partial \eta^n} \frac{\partial^n}{\partial \xi^n} - \frac{\partial^n h}{\partial \xi^n} \frac{\partial^n}{\partial \eta^n} \right\} x.$$

$$x(\eta, \xi, t) \quad ($$

A

$$\langle A \rangle = \int A(\xi, \eta) x^*(\eta, \xi, t) d\xi d\eta.$$

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