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## **Discontinuous Open Channel Flow Analysis with Finite Difference or Finite Element?**

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### **Abstract**

A discontinuous free-surface flow analysis in open channels using finite difference and finite element methods is presented. Difficulties related to routing discontinuous flow, such as hydraulic jumps, back water flow and flow in junctions, produces very poor results when finite difference and classical Galerkin finite element methods are applied. A variance of finite element method, called Petrov Galerkin, uses discontinuous weighted test functions to dampen the numerically generated oscillations. Here one-dimensional linear elements was shown to estimate the flow depth and velocity accurately.

**Key words:** Open channels, Discontinuous flow routing, Finite difference, Finite element

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 (nondissipative) (dispersive)  
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$$v \frac{\partial y}{\partial x} + y \frac{\partial v}{\partial x} + \frac{\partial y}{\partial t} = 0 \quad ( )$$

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} + g \left( \frac{\partial y}{\partial x} - S_o + S_f \right) = 0 \quad ( )$$

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[1] ( )

[3] ( )

$$S_o \quad v \quad y \quad S_f \quad g \quad [4] ( )$$

$t \quad x$

[5] ( )

$$S_f \quad S_o$$

$$\left( \frac{\partial y}{\partial x} \right)$$

$$\left[ +g(1-\theta)\frac{2\Delta t}{\Delta x}(y_{i+1}-y_i) + g(1-\theta)(2\Delta t)S_{fi} \right]^j - g(2\Delta t)S_0 = 0 \quad ( )$$

$\theta$

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$$\bar{y}_i = \frac{y_i + y_{i+1}}{2}, \quad \bar{v}_i = \frac{v_i + v_{i+1}}{2}$$

$$S_{fi} = \frac{n^2 v_i^2}{2.21} (1/R)^{4/3}$$

$S_{fi}$

$v_i$

$R$

$n$

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m-2

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$$C_i = \left[ (y_i + y_{i+1}) + \left(\theta \frac{2\Delta t}{\Delta x}\right)(\bar{v}_i)(y_{i+1} - y_i) \right]$$

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[8] ( )

$$\left[ +\theta \frac{2\Delta t}{\Delta x}(\bar{y}_i)(v_{i+1} - v_i) \right]^{j+1} + \left[ -(y_i + y_{i+1}) - \right]$$

$$(1-\theta)\left(\frac{2\Delta t}{\Delta x}\right)(\bar{v}_i)(y_{i+1} - y_i)$$

$$\left[ + (1-\theta)\frac{2\Delta t}{\Delta x}(\bar{y}_i)(v_{i+1} + v_i)^j = 0 \right] \quad ( )$$

$$M_i = \left[ (v_i + v_{i+1}) + \theta \frac{2\Delta t}{\Delta x}(\bar{v}_i)(v_{i+1} - v_i) \right]$$

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$$\left[ +g\theta \frac{2\Delta t}{\Delta x}(y_{i+1} + y_i) + g\theta(2\Delta t)S_{fi} \right]^{j+1}$$

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$$+ \left[ -(v_i + v_{i+1}) + (1-\theta)\left(\frac{2\Delta t}{\Delta x}\right)(\bar{v}_i)(v_{i+1} - v_i) \right]$$

$$R_2 = \frac{\partial \widehat{q}}{\partial t} + (c^2 - u^2) \frac{\partial \widehat{y}}{\partial x} + 2u \frac{\partial \widehat{q}}{\partial x} \quad ( ) \quad \text{(Petrov or Dissipative Galerkin)}$$

$$\begin{cases} \widehat{y} \\ \widehat{q} \end{cases} = \begin{bmatrix} N_1 & N_2 & 0 & 0 \\ 0 & 0 & N_1 & N_2 \end{bmatrix} \begin{cases} y_1 \\ y_2 \\ q_1 \\ q_2 \end{cases} \quad ( )$$

$$\frac{\partial q}{\partial x} + \frac{\partial y}{\partial t} = 0 \quad ( )$$

$$\frac{\partial q}{\partial t} + \frac{\partial}{\partial x} \left( \frac{q^2}{y} + \frac{gy^2}{2} \right) = 0 \quad ( )$$

$$c = \sqrt{gy} \quad u = \frac{q}{y}$$

$$[10] \quad \frac{\partial q}{\partial t} + (c^2 - u^2) \frac{\partial y}{\partial x} + 2u \frac{\partial q}{\partial x} = 0 \quad ( )$$

$$N_1^* = N_1 + \varepsilon \frac{\partial N_1}{\partial x} \quad ( ) \quad ( )$$

$$N_2^* = N_2 + \varepsilon \frac{\partial N_2}{\partial x} \quad ( )$$

$$\varepsilon \quad \frac{\partial U}{\partial t} + A \frac{\partial U}{\partial x} = 0 \quad ( )$$

$$\int_e N^{*T} \left( \frac{\partial U}{\partial t} + A \frac{\partial U}{\partial x} \right) dx = 0 \quad ( )$$

$$U = \begin{Bmatrix} y \\ q \end{Bmatrix}, \quad A = \begin{bmatrix} 0 & 1 \\ c^2 - u^2 & 2u \end{bmatrix}$$

$$N^{*T} \quad ( ) \quad ( )$$

$$N^* = N + \varepsilon A^T \frac{\partial N}{\partial x} \quad ( )$$

$$R_1 = \frac{\partial \widehat{y}}{\partial t} + \frac{\partial \widehat{q}}{\partial x} \quad ( )$$

$$N^* = \begin{bmatrix} N_1 & N_2 & \varepsilon(c^2 - u^2) \frac{\partial N_1}{\partial x} & \varepsilon(c^2 - u^2) \frac{\partial N_2}{\partial x} \\ \varepsilon \frac{\partial N_1}{\partial x} & \varepsilon \frac{\partial N_2}{\partial x} & N_1 + 2\varepsilon u \frac{\partial N_1}{\partial x} & N_2 + 2\varepsilon u \frac{\partial N_2}{\partial x} \end{bmatrix} \quad ( )$$

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$$M\dot{U} + CU = 0 \quad ( )$$

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M ( )

$\dot{U}$

U

u c

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$\varepsilon \Delta t$

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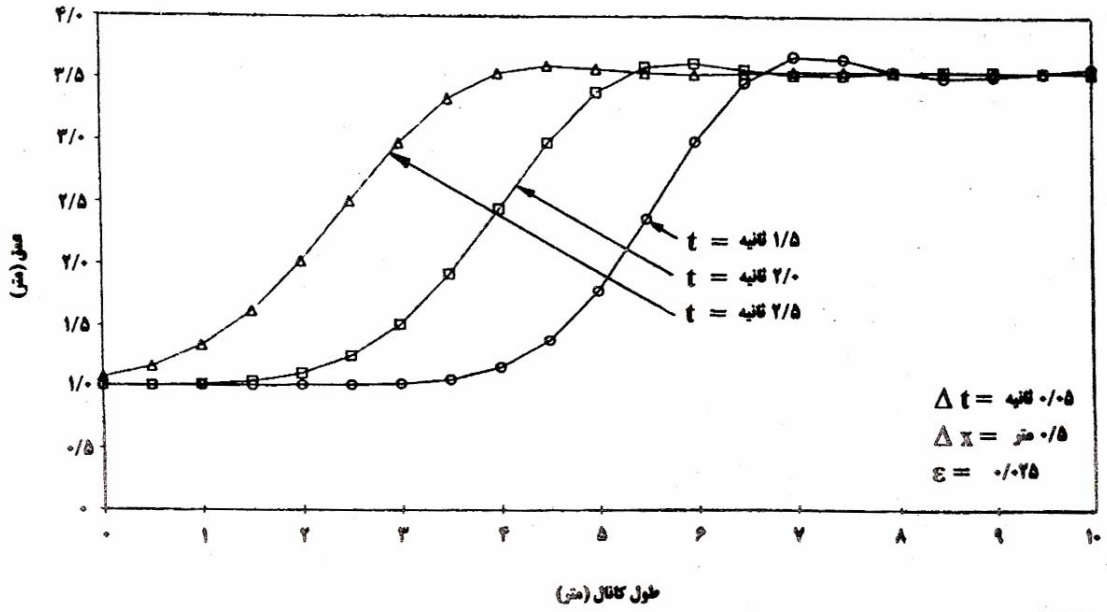
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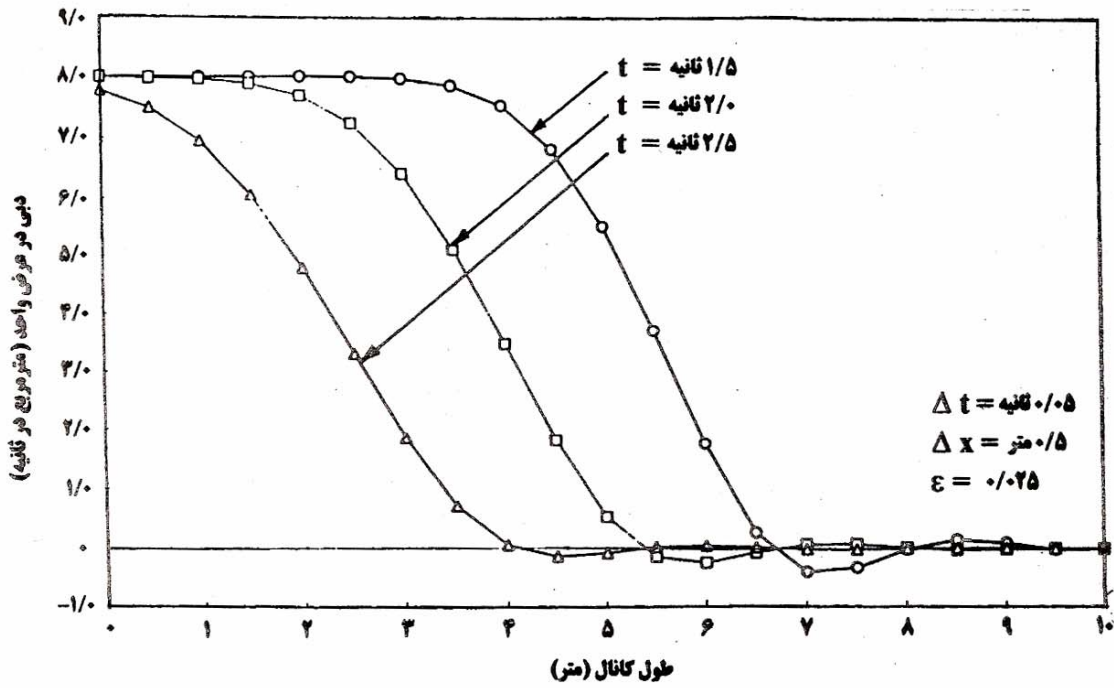
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( ) ( $\varepsilon = 0$ )

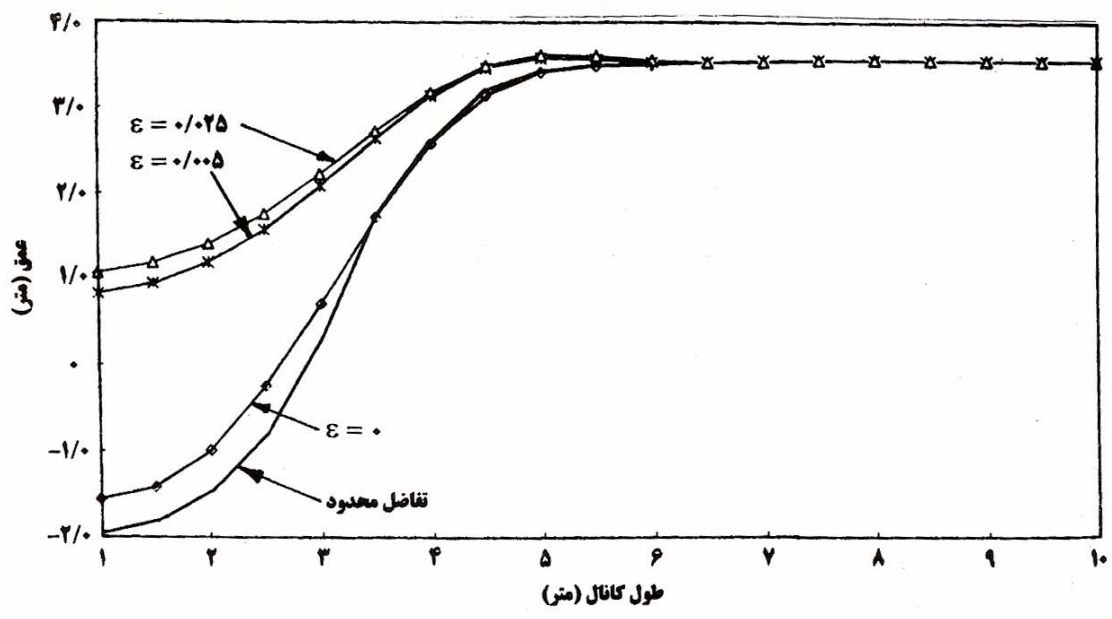


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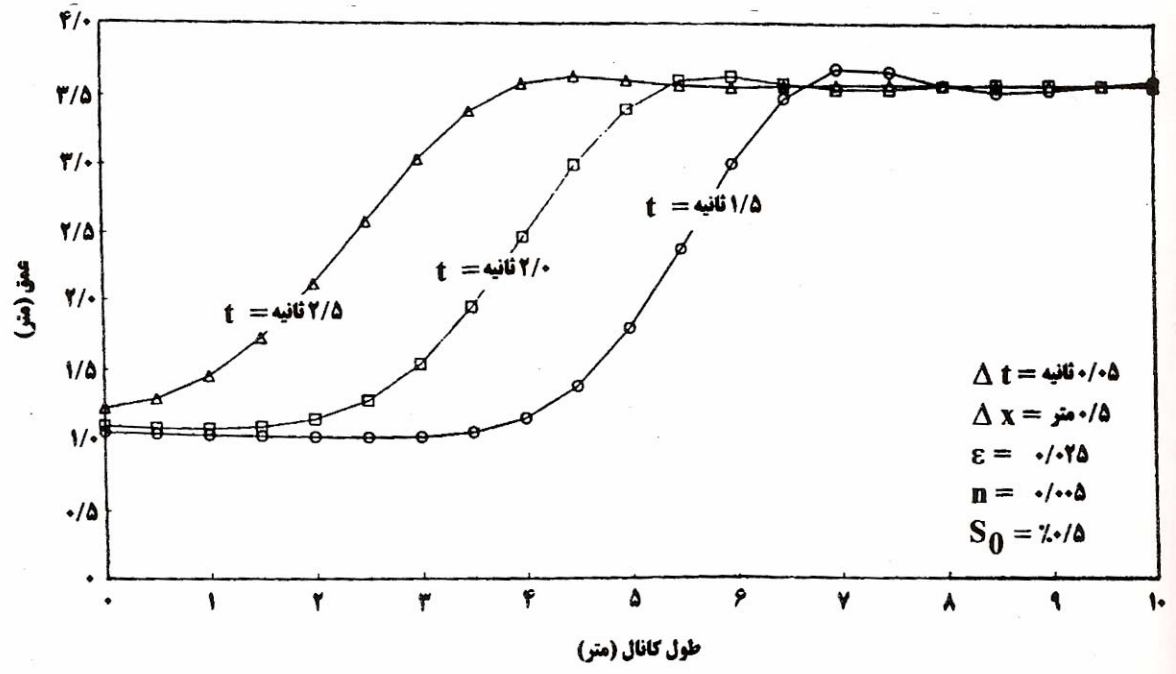


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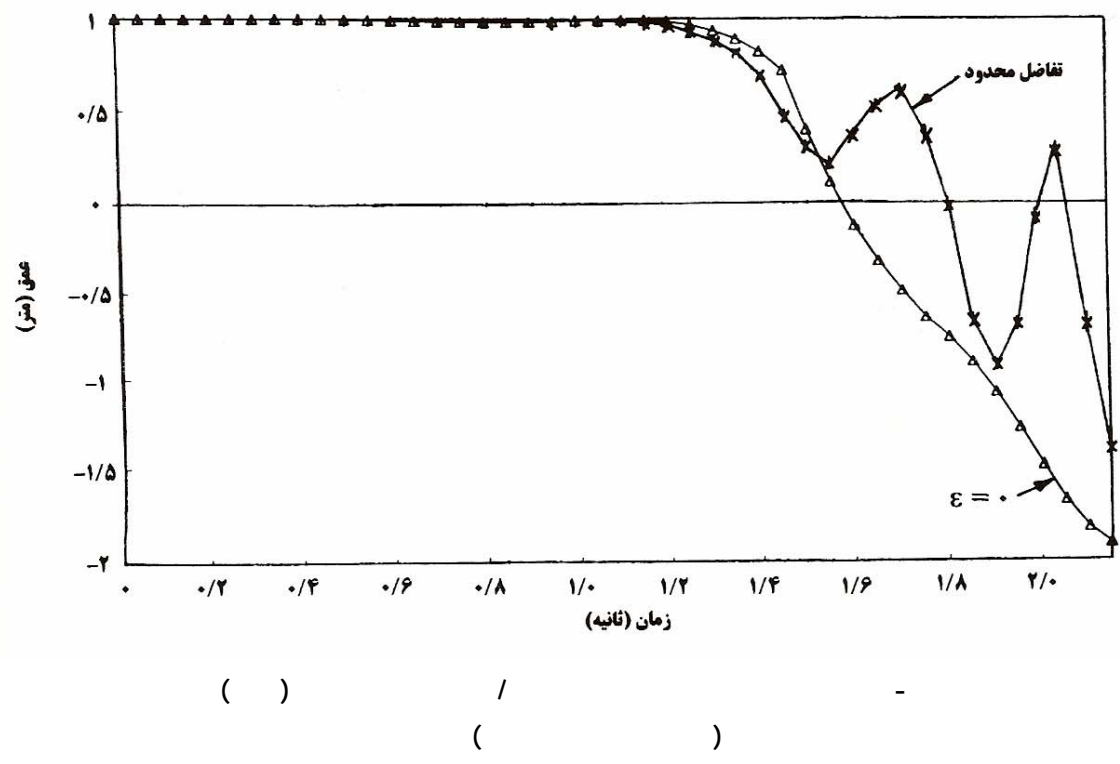
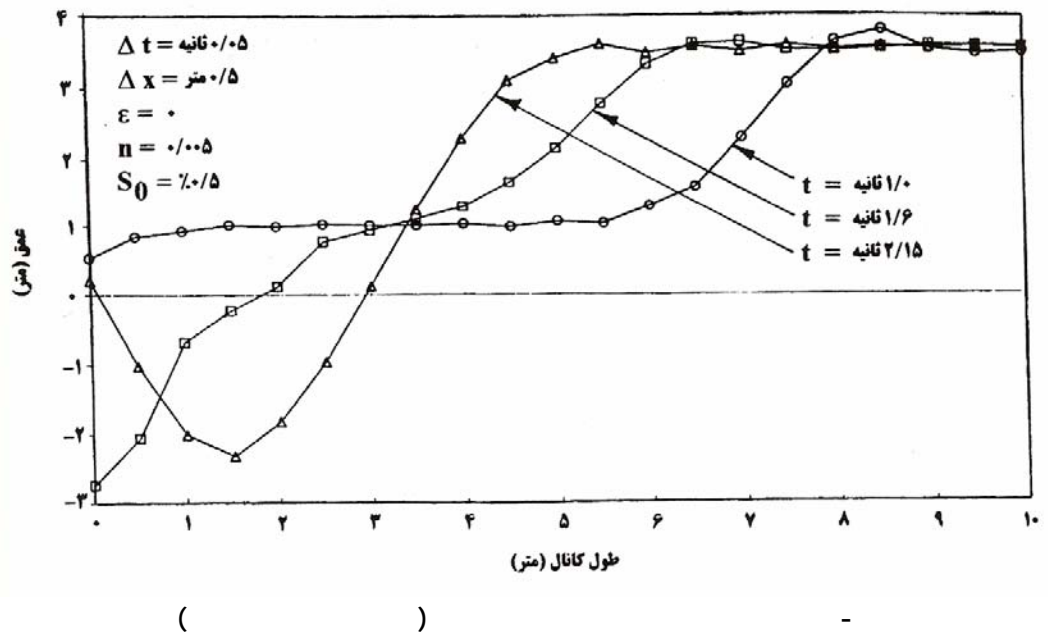
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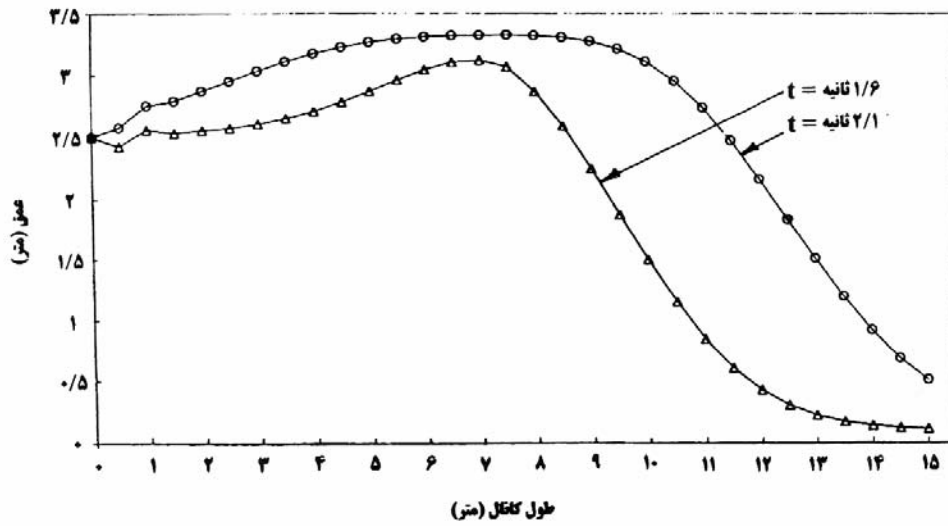
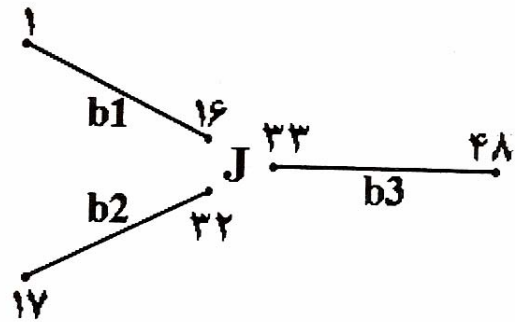


$$\begin{matrix}
 b_3 & & b_2 & b_1 & / \\
 & (b_3 & & ) & / \\
 & (b_2 & b_1 & ) & /
 \end{matrix}$$

$$\begin{aligned}
 y_{16} &= y_{32} = y_{33} \\
 q_{16} + q_{32} &= q_{33}
 \end{aligned}$$

$$\begin{matrix}
 b_2 & b_1 & / \\
 & ( & ) & /
 \end{matrix}$$

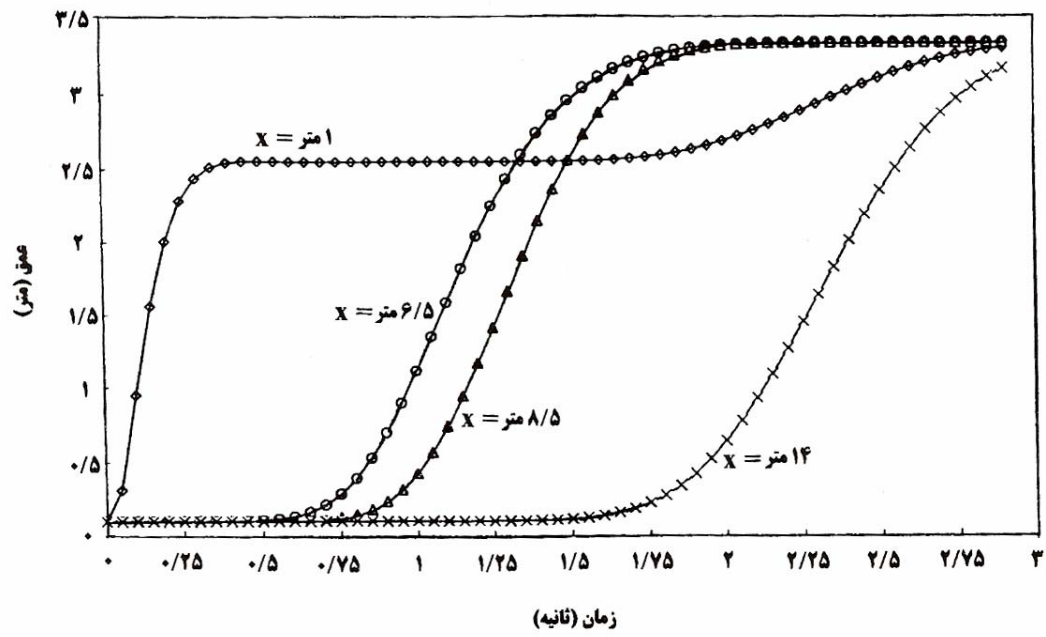
$$\begin{matrix}
 b_3 & b_1 & ( & ) \\
 & & / & /
 \end{matrix}$$



$$( \quad )$$

$$\begin{matrix}
 & & b_3 & & b_2 & b_1 & / \\
 / & ) & & ( & & & ( & ) \\
 & / & & ( & & & / & \\
 / & & ( & / & ) & & &
 \end{matrix}$$

$$b_2 \quad b_1$$



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b<sub>3</sub> b<sub>1</sub>

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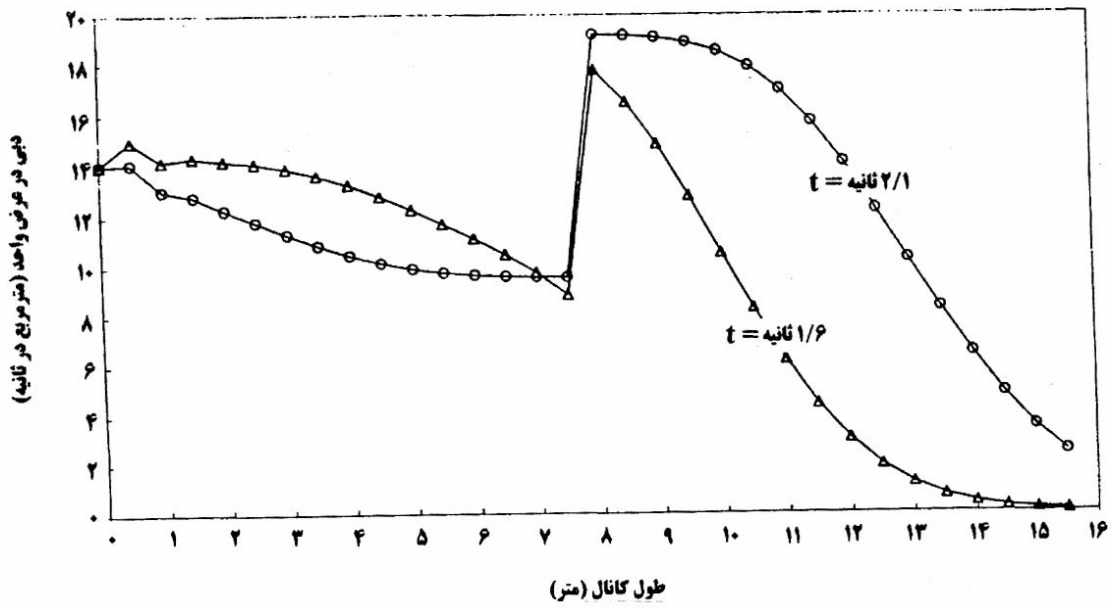
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b<sub>2</sub> b<sub>1</sub>

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