

ADI
(k-ε)

Simulation of Water Circulation in River Harbours Using the Zero- and Two-Equation Turbulence Models

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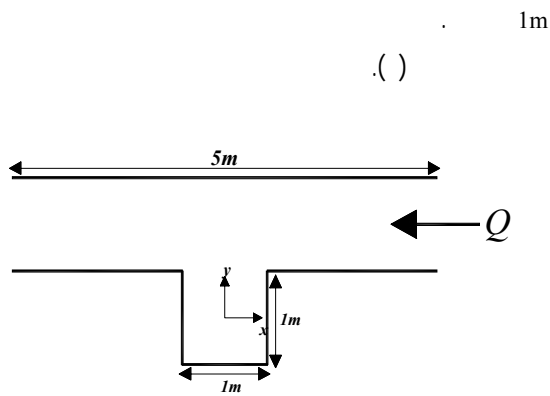
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Abstract

Details of a numerical model for simulation of the turbulent flow within a square river harbour are discussed. The turbulent flow has been modeled using the depth-integrated hydrodynamics Reynolds equations. The finite difference scheme has been used to discretize of the governing equations and they have been solved using the ADI method and Thomas algorithm. In the current study, we tried to apply the zero and two equation turbulence models (i.e. the mixing length and k-ε) and the different closed boundary conditions so that we can get the exact flow pattern within the harbour as far as possible.

Key words: Numerical Model, River Harbour, Flow Simulation, Turbulence Model, Closed Boundary Condition

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Jiang [] Ragab Jodan [] Langendoen
 Zhou [] Hosoda Kimura [] Nece [] Falconer []
 Van Schijndel [] Kranenburg

(Delft)

y x

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1m×1m

$$\frac{\partial \zeta}{\partial t} + \frac{\partial UH}{\partial x} + \frac{\partial VH}{\partial y} = 0 \quad ()$$

0.042 m³/s

5m

0.11m

$$4.05\text{mm} = k_s = R_e \left[\frac{\partial UH}{\partial t} + \beta \left[\frac{\partial U^2 H}{\partial x} + \frac{\partial UVH}{\partial y} \right] - f_c VH - gH \frac{\partial \zeta}{\partial x} + \frac{C_s \rho_a W_x W_s}{\rho} - \frac{gUV_s}{C^2} + \frac{\partial(-\overline{u'u'})H}{\partial x} + \frac{\partial(-\overline{u'v'})H}{\partial y} \right] \quad (1)$$

$$-\overline{u'_i u'_j} = \bar{v}_t \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) - \frac{2}{3} \bar{k} \delta_{ij} \quad (2)$$

$$= \bar{k} \quad = \bar{v}_t$$

$$= \delta_{ij} \quad i \neq j \quad i = j$$

$$\frac{\partial VH}{\partial t} + \beta \left[\frac{\partial UVH}{\partial x} + \frac{\partial V^2 H}{\partial y} \right] = -f_c UH - gH \frac{\partial \zeta}{\partial y} + \frac{C_s \rho_a W_y W_s}{\rho} - \frac{gVV_s}{C^2} + \frac{\partial(-\overline{v'u'})H}{\partial x} + \frac{\partial(-\overline{v'v'})H}{\partial y} \quad (3)$$

k-ε

$$= t = \zeta$$

$$= U, V = x, y$$

$$= H \quad y \quad x$$

$$= f_c = \beta$$

$$= \varpi = 2\varpi \sin \phi$$

$$= g \quad = \phi$$

$$= \rho_a = C_s$$

$$= W_x, W_y = \rho$$

$$= C = W_s \quad y \quad x$$

$$= -\overline{u'_i u'_j} = V_s$$

$$\bar{v}_t = 0.15 U_* H \quad (4)$$

$$= U_*$$

k-ε

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$$\frac{\partial \bar{k} H}{\partial t} + \frac{\partial \bar{k} U H}{\partial x} + \frac{\partial \bar{k} V H}{\partial y} = \frac{\partial}{\partial x} \left(\frac{\bar{v}_t H}{\sigma_k} \frac{\partial \bar{k}}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{\bar{v}_t H}{\sigma_k} \frac{\partial \bar{k}}{\partial y} \right) + \bar{v}_t H \left[2 \left(\frac{\partial U}{\partial x} \right)^2 + 2 \left(\frac{\partial V}{\partial y} \right)^2 + \left(\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} \right)^2 \right] + c_k U_*^3 - \bar{\epsilon} H \quad (5)$$

$$\beta = 1 + \frac{g}{\kappa^2 C^2} \quad (6)$$

$$.(=0.4) = \kappa$$

$$C = -\sqrt{32g} \log_{10} \left[\frac{k_s}{14.84H} + \frac{1.255C}{R_e \sqrt{2g}} \right] \quad (7)$$

(Thomas)

$$\frac{\partial \bar{\epsilon} H}{\partial t} + \frac{\partial \bar{\epsilon} U H}{\partial x} + \frac{\partial \bar{\epsilon} V H}{\partial y} =$$

$$\frac{\partial}{\partial x} \left(\frac{\bar{v}_t H}{\sigma_\epsilon} \frac{\partial \bar{\epsilon}}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{\bar{v}_t H}{\sigma_\epsilon} \frac{\partial \bar{\epsilon}}{\partial y} \right)$$

$$+ c_{1\epsilon} c_\mu \bar{k} H \left[2 \left(\frac{\partial U}{\partial x} \right)^2 + 2 \left(\frac{\partial V}{\partial y} \right)^2 + \left(\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} \right)^2 \right]$$

$$+ c_\epsilon \frac{U_*^4}{H} - c_{2\epsilon} \frac{\bar{\epsilon}^2}{\bar{k}} H \quad ()$$

$$= \bar{k}$$

$$= \sigma_k \sigma_\epsilon c_\mu c_{1\epsilon} c_{2\epsilon} = \bar{\epsilon}$$

$\sigma_\epsilon = 1.22$, $c_\mu = 0.09$, $c_{1\epsilon} = 1.44$, $c_{2\epsilon} = 1.92$

(= gC^{-2}) = C_f $\sigma_k = 1.0$

$c_\epsilon = 3.6 c_{2\epsilon} c_\mu^{1/2} C_f^{-3/4}$ $c_k = C_f^{-1/2}$

($Cr = \sqrt{gH\Delta t} / \Delta x$)

$$\bar{v}_t = c_\mu \frac{\bar{k}^2}{\bar{\epsilon}} \quad ()$$

(Third Order Upwind)

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x

$i+2$

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$$\left(\frac{\partial U q_x}{\partial x} \right)_{i+2} = \frac{(U q_x)_{i+5/2} - (U q_x)_{i+3/2}}{\Delta x} \quad ()$$

: $q_y = VH$ $q_x = UH$

$$q_x|_{i+5/2} = \lambda_1 (q_x|_{i+3} + q_x|_{i+2}) - \lambda_2 (q_x|_{i+3} - 2q_x|_{i+2} + q_x|_{i+1})$$

$$q_x|_{i+3/2} = \lambda_1 (q_x|_{i+2} + q_x|_{i+1}) - \lambda_2 (q_x|_{i+2} - 2q_x|_{i+1} + q_x|_i)$$

25mm*25mm

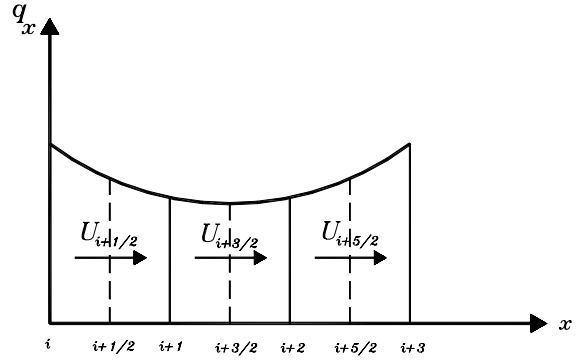
200*82

$\lambda_2 = 1/6$ $\lambda_1 = 1/2$ (Alternating Direction Implicit) ADI

$$\nabla^2 q'_x|_{i,j} = \begin{cases} q'_x|_{i+1/2,j} - 2q'_x|_{i-1/2,j} + q'_x|_{i-3/2,j} & \text{if } U \geq 0 \\ q'_x|_{i+3/2,j} - 2q'_x|_{i+1/2,j} + q'_x|_{i-1/2,j} & \text{if } U < 0 \end{cases}$$

$$\nabla^2 q'_y|_{i+1/2,j+1/2} = \begin{cases} q'_y|_{i+1,j+1/2} - 2q'_y|_{i,j+1/2} + q'_y|_{i-1,j+1/2} & \text{if } U \geq 0 \\ q'_y|_{i+2,j+1/2} - 2q'_y|_{i+1,j+1/2} + q'_y|_{i,j+1/2} & \text{if } U < 0 \end{cases}$$

$$\nabla^2 q'_y|_{i+1/2,j-1/2} = \begin{cases} q'_y|_{i+1,j-1/2} - 2q'_y|_{i,j-1/2} + q'_y|_{i-1,j-1/2} & \text{if } U \geq 0 \\ q'_y|_{i+2,j-1/2} - 2q'_y|_{i+1,j-1/2} + q'_y|_{i,j-1/2} & \text{if } U < 0 \end{cases}$$



$n - 1/2$

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$0.042\text{m}^3/\text{s}$

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(.)

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x

$$q_x|_{i+1/2,j}^{n+1/2} = q_x|_{i+1/2,j}^{n-1/2} - \frac{\beta\lambda_1\Delta t}{\Delta x} \left[\begin{aligned} & U'|_{i+1,j}^n (q'_x|_{i+3/2,j}^n + q'_x|_{i+1/2,j}^n) \\ & - U'|_{i,j}^n (q'_x|_{i+1/2,j}^n + q'_x|_{i-1/2,j}^n) \end{aligned} \right]$$

$$- \frac{\beta\lambda_1\Delta t}{\Delta x} \left[\begin{aligned} & U'|_{i+1/2,j+r}^n (q'_y|_{i+1,j+1/2}^n + q'_y|_{i+j+1/2}^n) \\ & - U'|_{i+1/2,j+s}^n (q'_y|_{i+1,j-1/2}^n + q'_y|_{i,j-1/2}^n) \end{aligned} \right]$$

$$+ \frac{\beta\lambda_2\Delta t}{\Delta x} \left[U'|_{i+1,j}^n \nabla^2 q'_x|_{i+1,j}^n - U'|_{i,j}^n \nabla^2 q'_x|_{i,j}^n \right]$$

$$+ \frac{\beta\lambda_2\Delta t}{\Delta x} \left[\begin{aligned} & U'|_{i+1/2,j+r}^n \nabla^2 q'_y|_{i+1/2,j+1/2}^n \\ & - U'|_{i+1/2,j+s}^n \nabla^2 q'_y|_{i+1/2,j-1/2}^n \end{aligned} \right]$$

$$- \frac{g\Delta t}{2\Delta x} H_x|_{i+1/2,j}^n [\zeta_{i+1,j}^{n+1/2} + \zeta_{i+1,j}^{n-1/2} - \zeta_{i,j}^{n+1/2} - \zeta_{i,j}^{n-1/2}]$$

$$- \Delta t \frac{g(U_{i+1/2,j}^{n+1/2} + U_{i+1/2,j}^{n-1/2})}{2C^2} \sqrt{U^2 + V^2}|_{i+1/2,j}^n$$

$$+ \frac{\Delta t}{\Delta x^2} \bar{v}_t H|_{i+1/2,j}^n \left[\begin{aligned} & U'_{i+3/2,j} + U'_{i-1/2,j} + U'_{i+1/2,j+1} \\ & + U'_{i+1/2,j-1} - 4U'_{i+1/2,j} \end{aligned} \right]$$

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$= \Delta x$

$= n$

$$s = (V/|V|)_{i+1/2,j-1/2} \quad r = (-V/|V|)_{i+1/2,j+1/2}$$

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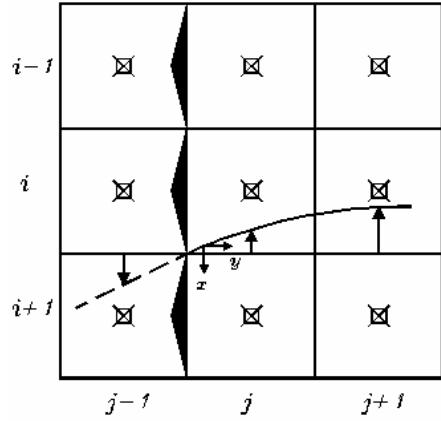
$$\nabla^2 q'_x|_{i+1,j} = \begin{cases} q'_x|_{i+3/2,j} - 2q'_x|_{i+1/2,j} + q'_x|_{i-1/2,j} & \text{if } U \geq 0 \\ q'_x|_{i+5/2,j} - 2q'_x|_{i+3/2,j} + q'_x|_{i+1/2,j} & \text{if } U < 0 \end{cases}$$

$$U_{i+1/2,j-1} \approx 0.916U_{i+1/2,j}$$

$$U(y) = U_{i+1/2,j+n} \left[\frac{y}{(n+1/2)\Delta y} \right]^{1/7} \quad ()$$

$$U_{i+1/2,j-1} = \alpha U_{i+1/2,j} \quad ()$$

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(fictitious point)

$$\frac{U_{i+1/2,j+1} - 2U_{i+1/2,j} + \alpha U_{i+1/2,j}}{\Delta y^2} = -\frac{6U_{i+1/2,j}}{49(\Delta y/2)^2} \quad ()$$

$$U_{i+1/2,j-1} = -U_{i+1/2,j} \quad ()$$

$$\alpha \approx 0.340$$

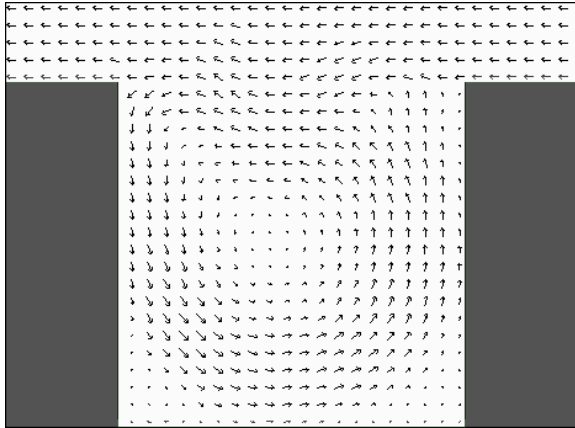
$$\tau_w = \frac{\rho g}{C^2} U^2 = \rho \nu_t \frac{\partial U}{\partial y} \quad ()$$

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$$\frac{\partial U}{\partial y} = \frac{\sqrt{g}}{C} \frac{U}{\sigma H} \quad ()$$

$$U_{i+1/2,j-1} = U_{i+1/2,j} \left[1 - \frac{\sqrt{g}}{C} \frac{\Delta y}{\sigma H} \right] \quad ()$$

$$= \sigma \quad = \Delta y$$



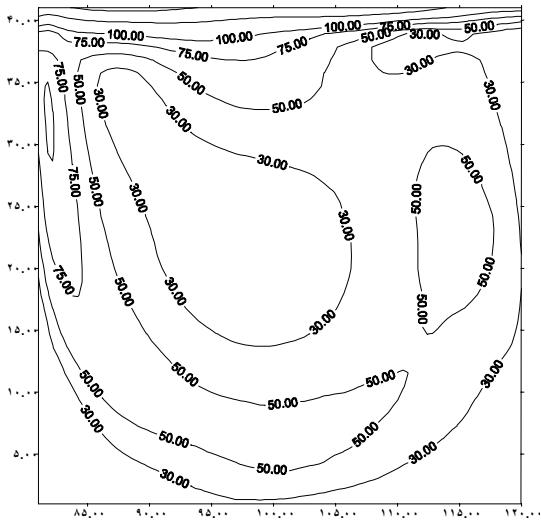
$(\bar{\varepsilon})$ (\bar{k}) k-ε

$$\bar{k}_w = \frac{U_*^2}{c_\mu^{1/2}} \quad ()$$

$$\bar{\varepsilon}_w = \frac{U_*^3}{KZ_c} \quad ()$$

= z_c

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(mm²/s)

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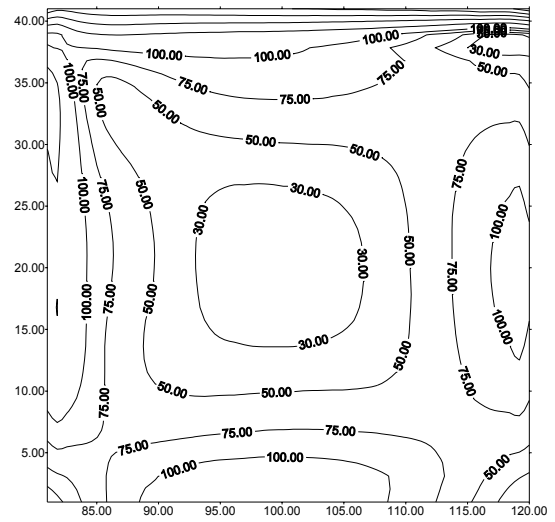
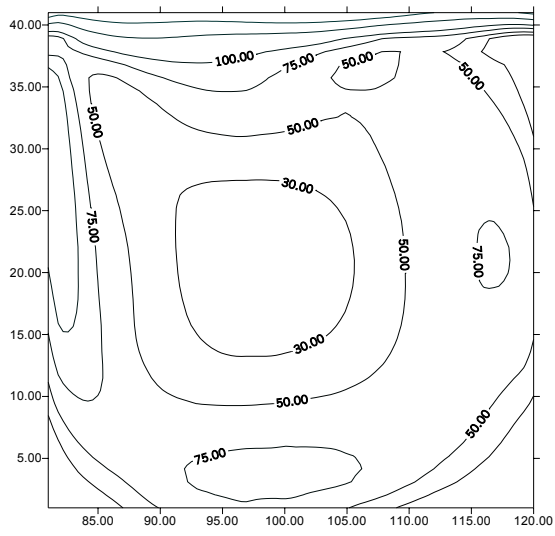
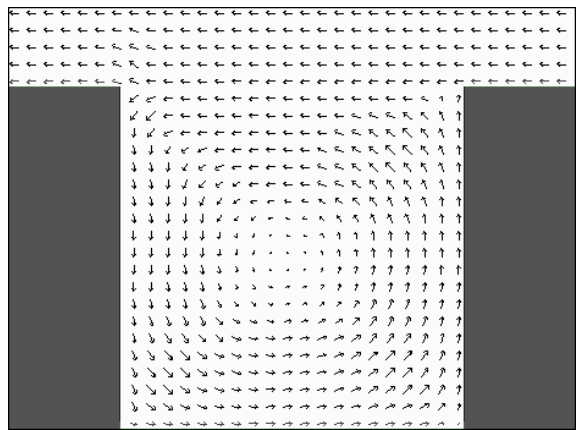
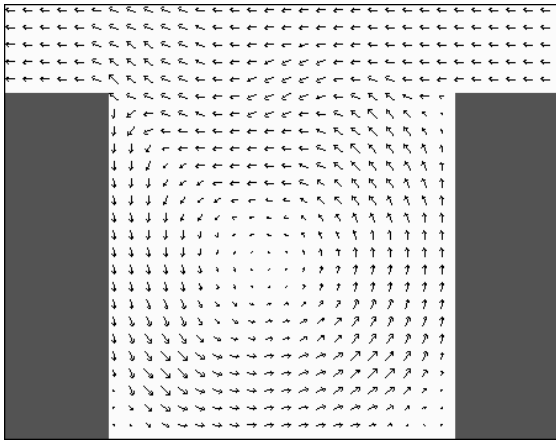
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(mm²/s)

(mm²/s)

k-ε

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k-ε

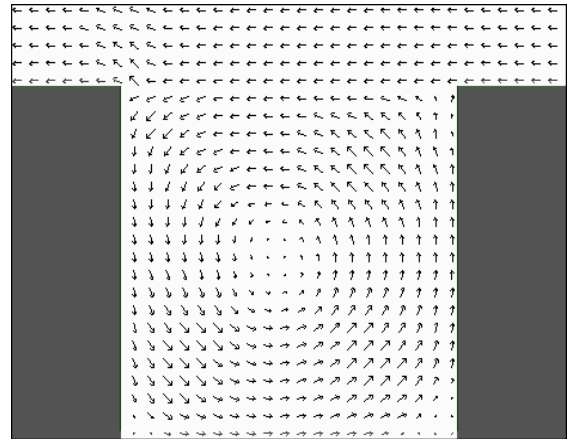
k-ε

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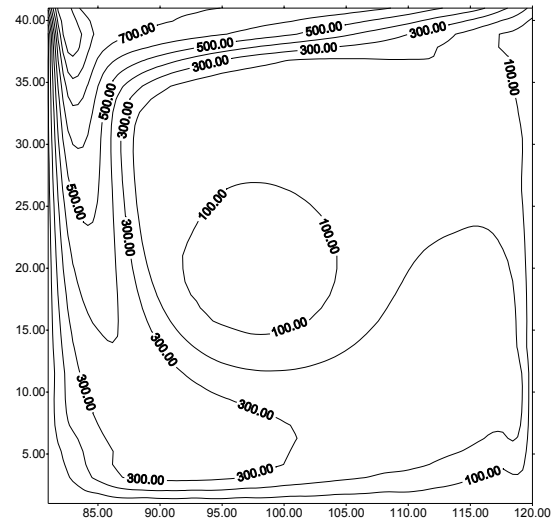
() () k-ε

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k-ε



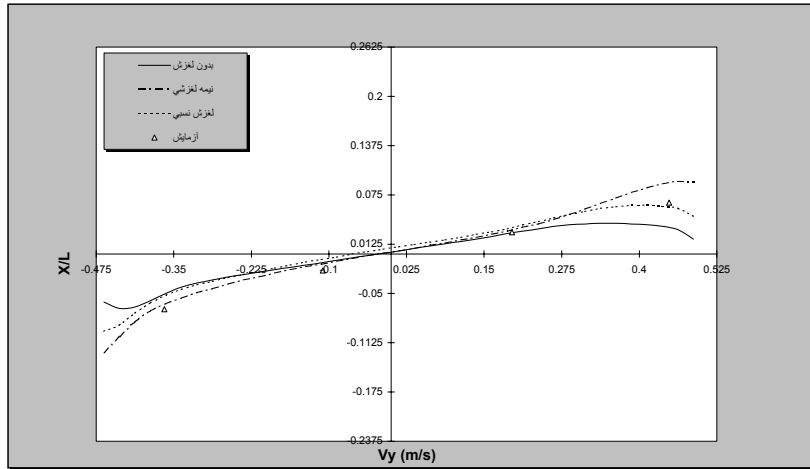
2.16m*0.54m

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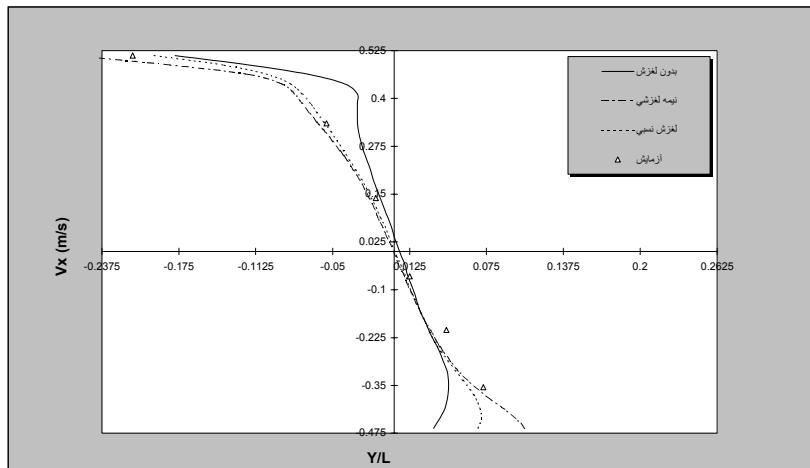
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 [] (0.54m*2.16m
 [] (1.08m*1.08m

(mm²/s)

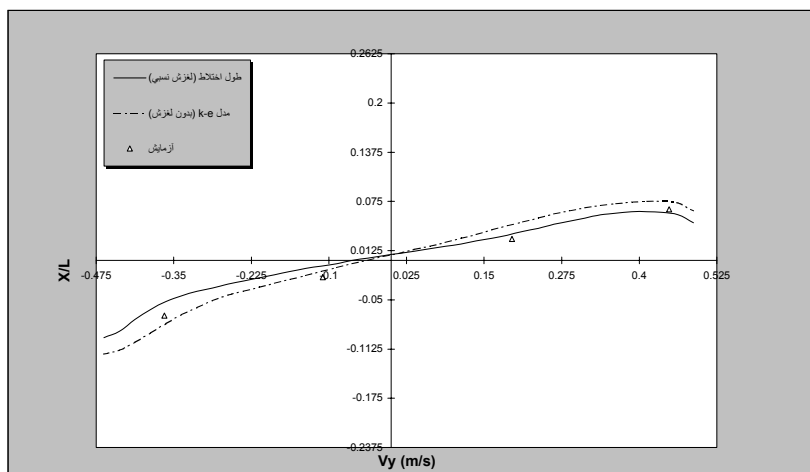
k-ε



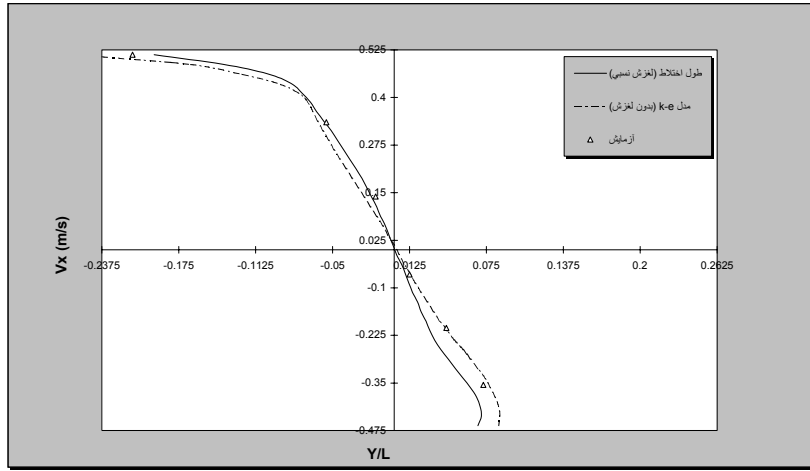
x



y



x



y

k-ε

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k-ε

k-ε

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k-ε

k-ε

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