

(Gallager) (LDPC: Low-Density Parity-Check) - -  
(Neal) (MacKay) - (Shannon)  
(SPA: Sum Product Algorithm)  
LDPC :

## A New Construction Method for Low-Density Parity-Check Codes

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### Abstract

Low-Density Parity-Check (LDPC) codes were introduced by Gallager in 1962. After more than three decades the near Shannon limit performance of these codes rediscovered by MacKay and Neal in 1996. Currently, LDPC code is one of the hot topics for research. In spite of simple iterative decoding for these codes by Sum-Product Algorithms (SPA), encoding and constructing of these codes have major difficulty. There exist a variety of methods for constructing LDPC codes. In this paper, we present and compare the most important of these methods. In addition we introduce a new constructing method to enhance the error performance.

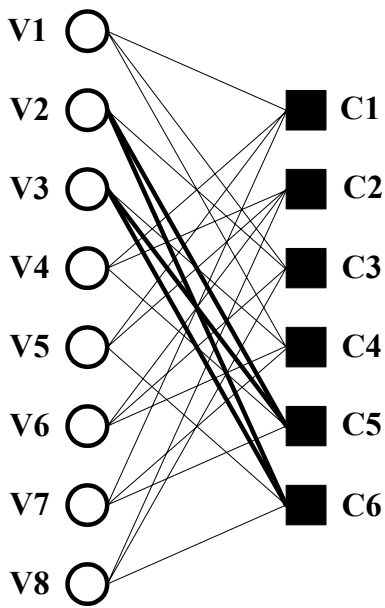
**Key words:** LDPC codes, Random parity-check matrix, Semi-random parity-check matrix, Error performance, Code construction.

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LDPC [ ] [ ] LDPC  
(ADSL: Asymmetric Digital Subscriber Line) [ ]  
[ ] (Tanner) [ ] expander  
LDPC [ ] GF (q) [ ]  
(BSC: Binary Symmetric Channel) LDPC (AWGN: Additive White Gaussian Noise)  
LDPC :  
[ ] [ ] (Poisson) [ ]  
(Sub-Poisson) - (Super-Poisson) - [ ] (Interleavers)  
0.05dB ) (Error Floor) (  
LDPC -  
[ ] - [ ] : [ ]  
[ ] - [ ]  
LDPC -  
[ ] LDPC  
0.0045dB : [ ] LDPC  
LDPC [ ] - LDPC  
[ ]

$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$



LDPC

LDPC

(LDPC: Low-Density Parity-Check)

LDPC

(BG: Bipartite Graph)

M LDPC H

N

N

M ( ) (Variable Node)

( ) (Check Node)

$V_i$  LDPC  $-( )$

$C_i$  H "1"

LDPC

$-(dv,dc)$

(Girth)

$( )$   $( )$

$dc$   $dv$

$dv$  H

LDPC  $dc$

$-( )$

LDPC

$( )$

$[ ]$

$[ ]$

(Soft)  $y_n = (1 - 2v_n) + w_n$   
 $w_n = AWGN \sim (0, \sigma^2)$  :

**(BP: Belief Propagation)**

**LDPC**

BP LDPC

[ ] (Pearl)

[ ] -

$q_{mn}^z = P_n^z = P(v_n = z)$  ( )

[ ]

LOG-BP

BP

[ ]

(Message-Passing)

( m )

[ ]

(Factor Graph)

$r_{mn}^z = \frac{1 + (-1)^z \prod_{N(m) \setminus n} (q_{mn}^0 - q_{mn}^1)}{2}$  ( )

H

m

n

$N(m) = \{n: H_{mn} = 1\}$

( n )

n

m

$q_{mn}^z = \alpha_{mn} P_n^z \prod_{M(n) \setminus m} r_{mn}^z$  ( )

$N(m)$

$M(n) = \{m: H_{mn} = 1\}$

$M(n) \setminus m$

m

$M(n)$

$N(m) \setminus n$

n

BP

$r_{mn}^z$

$q_{mn}^z$

z

H

$q_{mn}^z$

(PPP: Pseudo-Posteriori Probability)

$q_n^z$  :

z n

$(M(n) \setminus m)$

m

$q_n^z = \alpha_n P_n^z \prod_{M(n)} r_{mn}^z$  ( )

z n

m

$r_{mn}^z$

$\{q_{mk} : k \in$

$P(v_n = z) \cdot N(m) \setminus n\}$

$q_n^0 + q_n^1 = 1$  :  $\alpha_n$

AWGN

BPSK

$\hat{V} = [\hat{v}_n]$  ,  $\hat{v}_n = \underset{z}{\text{ArgMax}} P_n^z \prod_{M(n)} r_{mn}^z$  ( )

$P(y_n = z) = \frac{1}{1 + e^{2(2z-1)y_n/\sigma^2}}$  ( )

: PPP

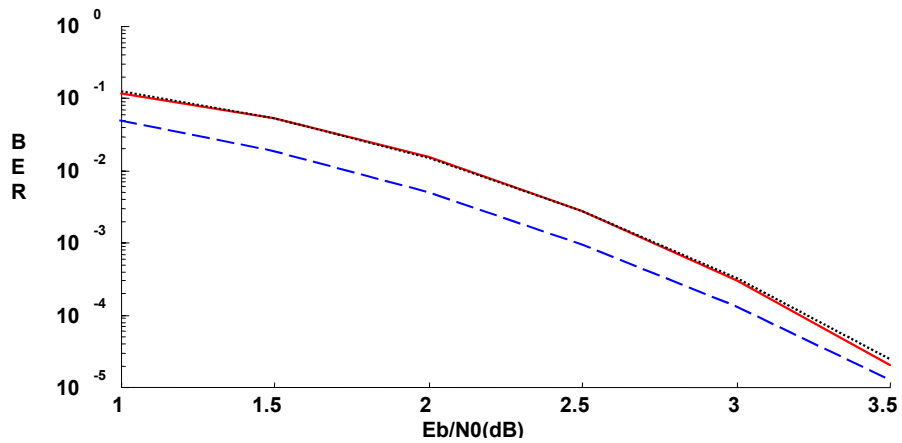


dv [ ]

dv [ ]

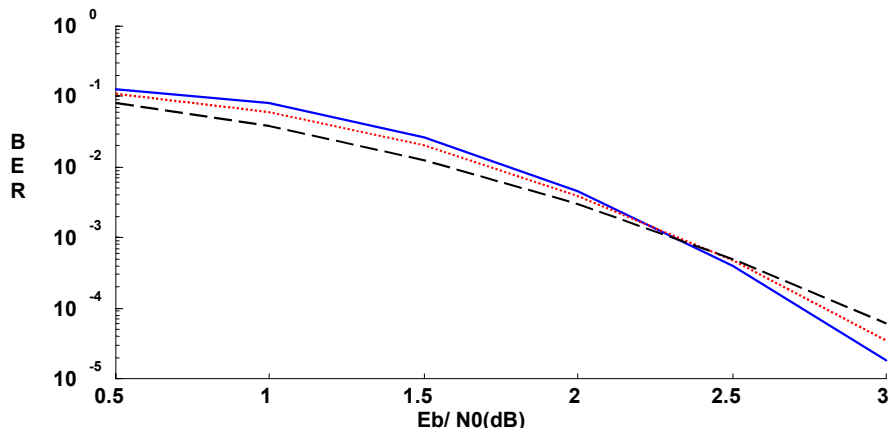
( )

( )



- :Dash ( ) - :Dot ( ) :Solid .

/ .( )



:Dash ( ) - :Dot ( ) :Solid .

/ .( )

Eb/N0

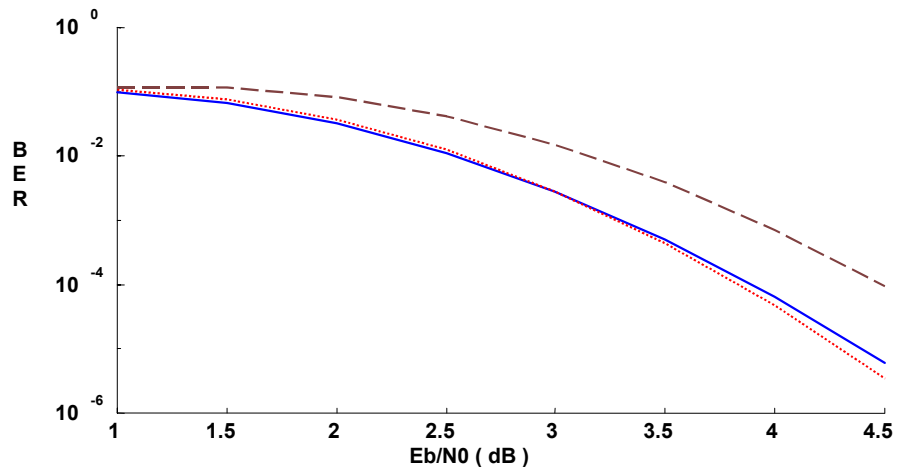
dB

( )

dv

$$\frac{E_b}{N_0} = \frac{1}{2R\sigma^2}$$

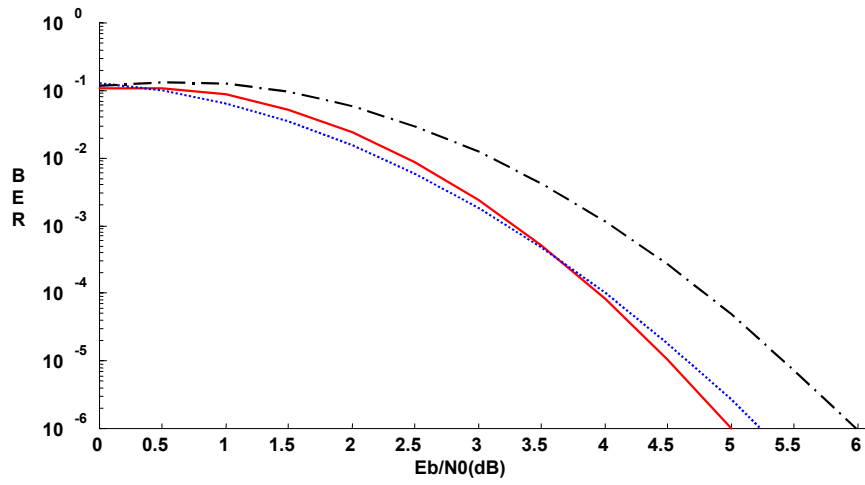
( )



- :Dash ( ) - :Dot ( ) :Solid .  
/ .( )

C(N,dv,dc,dd) - -  
dd C(N,dv,dc) ( )  
M×dd  
C(N,dv,dc) -  
dv -  
dv M

....  
( N=180 R=11/18 K=110  
70\*180 -  
dd<1 [ ] dv  
dd 0.1<dd<0.9  
0.9 0.1 dc=N.dv/M ) 7 dv  
( ) H ( )  
- 1dB  
(0.2dB ) ( )



(Solid) ( / ) (Dash-Dot) ( ) (Dot) dB  
 dv . ( ) -  
 ( / ) /

- H  
 N  
 K  
 M  
 / H dv  
 / H dc  
 R  
 V  
 S  
 H dd  
 $\sigma$

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