

## **Circuit Modeling and Measurement of Noise for a Semiconductor Laser Diode**

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### **Abstract**

This paper presents equivalent circuit models for both relative intensity noise (RIN) and phase/frequency noise spectrum (FNS) in a single semiconductor laser diode. The model for the electrical phase noise of a single mode laser is proposed for the first time. These equivalent circuit models are derived from the rate equations including the Langevin noise sources. Then, RIN and FNS are calculated in terms of electrical parameters. Finally, we explain an indirect experimental method used to measure RIN and FNS of a typical optical communication laser diode. Behavior of the experimental results is in agreement with those calculated by circuit models.

**Key words:** Relative intensity noise (RIN), Frequency/ phase noise spectrum (FNS), Equivalent circuit modeling, Semiconductor laser, Mode-Hopping.

... /

$$\frac{dN}{dt} = \frac{I}{q} - \gamma_e N - GP + F_N(t) \quad ( - )$$

(<sup>( )</sup>FNS) (<sup>( )</sup>RIN)

$$\frac{dP}{dt} = (G - \gamma)P + R_{sp} + F_P(t) \quad ( - )$$

( ) RIN

$$\frac{d\phi}{dt} = -(\omega - \omega_{th}) + \frac{1}{2}\beta_c(G - \gamma) + F_\phi(t) \quad ( - )$$

FNS

$q \ I \ \phi \ P \ N$

$R_{sp}$

[ ]

$\omega_{th} \ \omega$

RIN

$F_P(t) \ F_N(t)$

$\beta_c$

[ ]

$F_\phi(t)$

FNS

RIN

[ ]

$\tau_p \ \gamma_e = 1/\tau_e \ \gamma = 1/\tau_p$

$G$

$\tau_e$

$$G = A(N - N_0)(1 - P/S_p) \quad ( )$$

$N_0$

$A$

RF

$S_p$

[ ]

FNS RIN

$$\frac{\bar{I}}{q} - \gamma_e \bar{N} - \bar{G}\bar{P} = 0 \quad ( - )$$

[ - ]

FNS RIN

$$(\bar{G} - \gamma)\bar{P} + R_{sp} = 0 \quad ( - )$$

$\bar{G} \ \bar{I} \ \bar{P} \ \bar{N}$

( )

( )

[ ]

$$\langle F_j(t) \rangle = 0 \quad ( - )$$

$$\langle F_k(t)F_j(t') \rangle = 2D_{kj}\delta(t-t') \quad ( - )$$

- 
- 1- Relative Intensity Noise
  - 2- Frequency Noise Spectrum
  - 3- Mode Hopping
  - 4- Langevin rate equations

$$\delta \tilde{\phi} = \left[ \frac{1}{2} A \beta_c \delta \tilde{N} + \tilde{F}_\phi \right] / j \omega \quad ( - ) \quad : \quad D_{kj}$$

$$D = -\omega^2 + j \omega (\Gamma_N + \Gamma_P) + \Gamma_N \Gamma_P + A \bar{P} \bar{G}_l \quad ( - )$$

$$D_{NN} = R_{sp} \bar{P} + \gamma_e \bar{N}; \quad D_{PP} = R_{sp} \bar{P}; \quad ( - )$$

$$D_{\phi\phi} = \frac{R_{sp}}{4\bar{P}}; \quad D_{NP} = -R_{sp} \bar{P}; \quad D_{N\phi} = D_{P\phi} = 0$$

$\delta V$

[ ]

$\bar{P} \quad \bar{N}$

$$\delta V = m V_T \frac{\delta N}{N}; \quad ( - ) \quad [ ]$$

$$V_T = kT / q$$

$$m = 2 + \frac{\bar{N}}{\text{Vol} 2\sqrt{2}} \left[ \frac{1}{N_v} + \frac{1}{N_c} \right] \quad ( - )$$

$\delta P$

$\delta N$

$\delta \phi$

$V_T$

$N_c \quad N_v$   
 $\text{Vol}$

$$\frac{d \delta N(t)}{dt} = \frac{\delta I(t)}{q} - \Gamma_N \delta N(t) - \bar{G}_l \delta P(t) + F_N(t) \quad ( - )$$

$$i_L = q G_l \delta P \quad ( - )$$

$$\frac{d \delta P(t)}{dt} = A \bar{P} \delta N(t) - \Gamma_P \delta P(t) + F_P(t) \quad ( - )$$

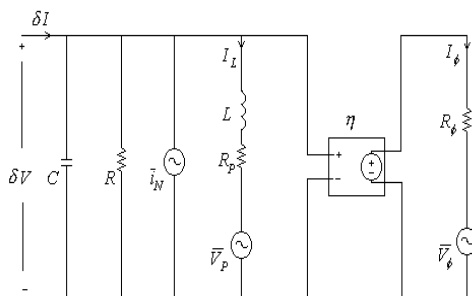
$$i_\phi = \omega \delta \phi \quad ( - )$$

$$\frac{d \delta \phi(t)}{dt} = \frac{1}{2} A \beta_c \delta N(t) - \frac{1}{2} \beta_c \frac{\bar{G}_{nl}}{\bar{P}} \delta P(t) + F_\phi(t) \quad ( - )$$

FNS

[ ] [ ]

RIN



$$\Gamma_N = \gamma_e + A \bar{P} \quad ( - )$$

$$\Gamma_P = R_{sp} / \bar{P} + 2 \bar{G}_{nl} \quad ( - )$$

: ( )

$$\delta \tilde{N} = [(j\omega + \Gamma_P) \tilde{F}_N - \bar{G}_l \tilde{F}_P] / D \quad ( - )$$

$$\delta \tilde{P} = [A \bar{P} \tilde{F}_N + (j\omega + \Gamma_N) \tilde{F}_P] / D \quad ( - )$$

$$S_p(\omega) \quad : \quad ( )$$

( )

$$f \ll f_r$$

$$f_r$$

RIN

$$C \frac{d \delta V}{d t} = \delta I - \frac{\delta V}{R} - i_L + i_N \quad ( - )$$

$$L \frac{d i_L}{d t} = \delta V - R_p i_L - V_p \quad ( - )$$

$$R_\phi i_\phi = \eta \delta V - V_\phi \quad ( - )$$

1.3 μm	λ	
250 μm	L	
2 μm	W	
0.2 μm	d	
0.1 ps	τ	
10 <sup>8</sup>	N <sub>0</sub>	
3.3×10 <sup>9</sup>	S <sub>p</sub>	
1.7×10 <sup>12</sup> s <sup>-1</sup>	R <sub>sp</sub>	
1.6 ps	τ <sub>p</sub>	
2.2 ns	τ <sub>e</sub>	
5625 s <sup>-1</sup>	A	
5	β <sub>c</sub>	

$$R \quad C$$

$$I_\phi \quad i_L$$

$$( ) \quad R_p$$

$$: \quad ( )$$

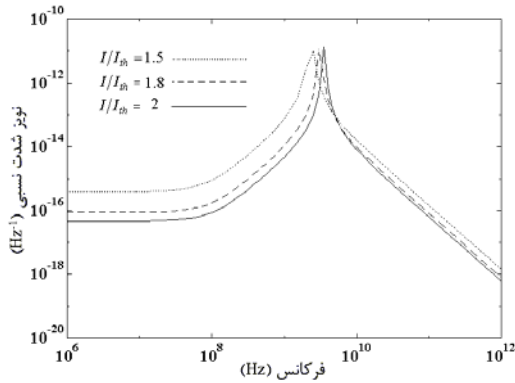
$$C = \frac{q \bar{N}}{m V_T}; \quad R = \frac{1}{C \Gamma_N}; \quad L = \frac{1}{C A \bar{G}_l \bar{P}};$$

$$R_p = L \Gamma_p; \quad R_\phi = \frac{2 q \eta}{A C \beta_c}; \quad \eta = A^3 \quad ( )$$

$$S_N = \frac{\bar{i}_N^2}{\Delta f} = 2 q^2 (\gamma_e \bar{N} + R_{sp} \bar{P}) \quad ( - )$$

$$S_p = \frac{\bar{V}_p^2}{\Delta f} = 2 (q L \bar{G}_l)^2 R_{sp} \bar{P} \quad ( - )$$

$$S_\phi = \frac{\bar{V}_\phi^2}{\Delta f} = R_\phi^2 \frac{R_{sp}}{2 \bar{P}} \quad ( - )$$

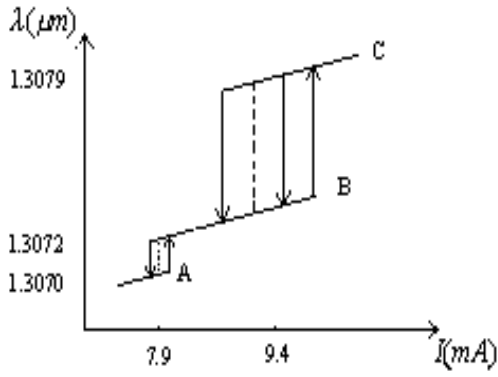


$$RIN(\omega) = \frac{S_p(\omega)}{\bar{P}^2} = \frac{\bar{i}_L^2(\omega)}{(I - I_{th})^2} \quad ( )$$

MHN

C B

RIN



.FLD3C5LK

(C B )

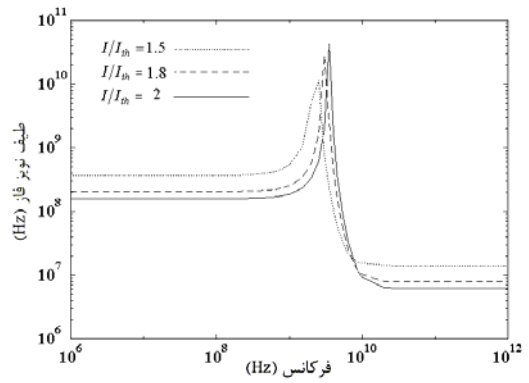
(A )

MHN

$$S_{\phi}(\omega) = \left\langle \left| \omega \delta \tilde{\phi}(\omega) \right|^2 \right\rangle = \bar{i}_{\phi}^{-2} \quad ( )$$

FNS ( )

RIN FNS



FNS RIN

MHN

$$N_0 \beta_c A$$

FNS RIN (MHN)

(FLD3C5LK)

( )

$\Delta f$

MHN / mA

$f_0$

$P_{out}$

[ ] [ ]

FNS RIN

[ ]

MHN

( )

FLD3C5LK

[ ]

$\omega = 0$

(C B ) / mA (A ) / mA

/ mA

$$\bar{P} = \frac{R_{sp}}{4\pi\Delta f} (1 + \beta_c^2)$$

( )

$$f_r = \omega_r / 2\pi; \quad \omega_r = \left( \frac{A(I - I_{th})}{q} \right)^{1/2} \quad ( )$$

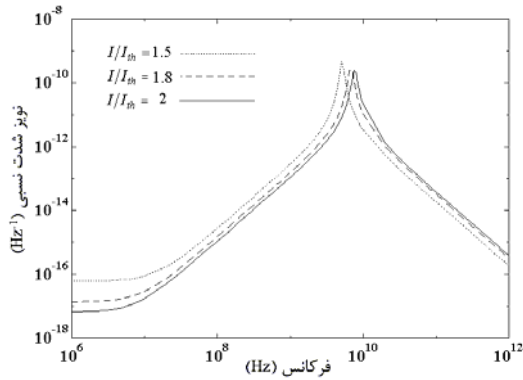
$$\Gamma_R = \frac{1}{2}(\Gamma_N + \Gamma_P), \quad ( )$$

( - ) ( - )  $\Gamma_P$   $\Gamma_N$   
( ) ( )

$$RIN = 2R_{sp} \times \left\{ \frac{(\Gamma_N^2 + \omega^2) + A^2 \bar{P}^2 (1 + \gamma_e \bar{N} / R_{sp} \bar{P}) - 2A\Gamma_N \bar{P}}{\bar{P}[(\omega_r - \omega)^2 + \Gamma_R^2][(\omega_r + \omega)^2 + \Gamma_R^2]} \right\} \quad ( )$$

$$S_{\phi}(\omega) \cong \frac{R_{sp}}{2\bar{P}} \times \left\{ 1 + \left\{ \beta_c^2 \omega_r^4 / [(\omega_r - \omega)^2 + (2\omega\Gamma_R)^2] \right\} \right\} \quad ( )$$

FNS RIN ( ) ( )



FLD3C5LK

FNS RIN

$\bar{P}$   $R_{sp}$

( ) -

$\gamma_e$

$$\bar{N} = \frac{I/q + AN_0 \bar{P}}{\gamma_e + A \bar{P}} \quad ( )$$

[ ]

$$\gamma_e = I_{th} / q \bar{N} \quad ( )$$

$\gamma_e$

$\bar{N}$  ( )

$\bar{N}$   $\lambda_e$

$R_{sp}$

[ ]

$$R_{sp} = \beta \bar{N} / \tau_e = \beta \bar{N} \gamma_e \quad ( - )$$

$$\beta = \frac{\zeta k A (\bar{N} - N_0)}{\gamma_e \bar{N} (E_f - h f_0)} T \quad ( - )$$

$$E_f \quad \zeta \sim 1.4 \quad 10^{-5} < \beta < 10^{-3}$$

( )

$R_{sp}$

$$\gamma_e \bar{N} \bar{P} R_{sp} \quad ( ) ( )$$

FNS RIN

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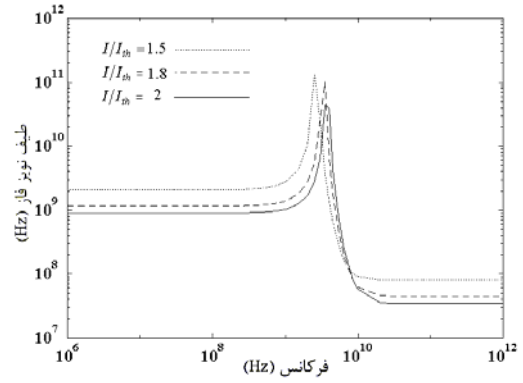
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FNS RIN

FNS RIN

FLD3C5LK

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