

Numerical modeling for flow in open channel with surface tracking method

M. Goodarzi Department of Mechanical Engineering, Faculty of Engineering, Bu Ali Sina University, Hamadan, Iran
A.R. Azimian Department of Mechanical Engineering, Isfahan University of Technology, Isfahan, Iran

Abstract

In this paper a numerical model has been introduced to solve the flow in open channels. The algorithm of this numerical model was developed in such a manner that prevents the instabilities, which are produced by the surface waves. With this model, the effects of shear stresses on the variations of the free surface elevation, especially in the regions of stronger shear stresses, were studied for three different open channels. Also the effect of Froud number increasing was studied. The numerical results showed that by increasing the Froud number the accuracy of the model was reduced. The details of numerical procedure were studied in a case, which convergence was failed.

Key words: Free surface flow, Froud number, Non-hydrostatic pressure.

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(x,y,z)

z

xy

G

$$G = \mu_t (u_{i,j}^j + u_{j,i}^i) u_{i,j}^i, \quad (1)$$

$i=1,2,3, \quad j=1,2,3$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0.0 \quad (2)$$

$$\rho(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}) - \mu_{eq} (\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}) = -\frac{\partial P}{\partial x} \quad (3)$$

$$\mu_t = \rho c_\mu \frac{k^2}{\varepsilon} \quad (4)$$

$$\rho(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z}) - \mu_{eq} (\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}) = -\frac{\partial P}{\partial y} \quad (5)$$

$k-\varepsilon$ σ_ε σ_k c_μ c_2 c_1
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$$\rho(u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z}) - \mu_{eq} (\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}) = -\frac{\partial P}{\partial z} - \rho g \quad (6)$$

[] $k-\varepsilon$ -

c_μ	c_1	c_2	σ_k	σ_ε
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$k - \varepsilon$

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$$\rho(u \frac{\partial k}{\partial x} + v \frac{\partial k}{\partial y} + w \frac{\partial k}{\partial z}) - \frac{\mu_t}{\sigma_k} (\frac{\partial^2 k}{\partial x^2} + \frac{\partial^2 k}{\partial y^2} + \frac{\partial^2 k}{\partial z^2}) = G - \rho \varepsilon \quad (7)$$

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$$\rho(u \frac{\partial \varepsilon}{\partial x} + v \frac{\partial \varepsilon}{\partial y} + w \frac{\partial \varepsilon}{\partial z}) - \frac{\mu_t}{\sigma_\varepsilon} (\frac{\partial^2 \varepsilon}{\partial x^2} + \frac{\partial^2 \varepsilon}{\partial y^2} + \frac{\partial^2 \varepsilon}{\partial z^2}) = c_1 \frac{\varepsilon}{k} G - \rho c_2 \frac{\varepsilon^2}{k} \quad (8)$$

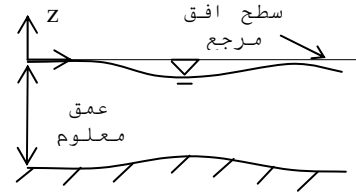
(ψ)

$$\psi = P + \rho g z - P_{am} \quad (9)$$

$w \quad v \quad u$
 $\rho \quad p \quad z \quad y \quad x$
 $\mu \quad g$

1- non-hydrostatic pressure

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$$\rho(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}) -$$

$$\mu_{eq} (\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}) = - \frac{\partial \psi}{\partial x} \quad ()$$

$$\rho(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z}) -$$

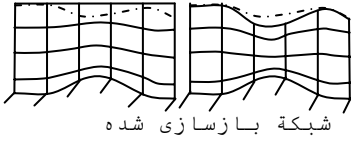
$$\mu_{eq} (\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}) = - \frac{\partial \psi}{\partial y} \quad ()$$

$$\rho(u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z}) -$$

$$\mu_{eq} (\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}) = - \frac{\partial \psi}{\partial z} \quad ()$$

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$$z_{fs} = z_{fs}(x, y) \quad ()$$

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w

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$$w_{fs} = \frac{Dz_{fs}}{Dt} = u_{fs} \frac{\partial z_{fs}}{\partial x} + v_{fs} \frac{\partial z_{fs}}{\partial y} \quad ()$$

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$$(P = P_{atm})$$

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$$z_{fs} = \frac{\psi_{fs}}{\rho g}$$

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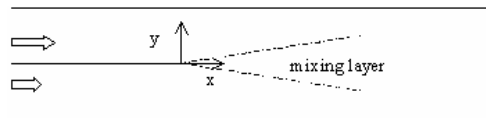
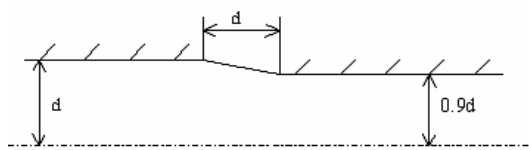
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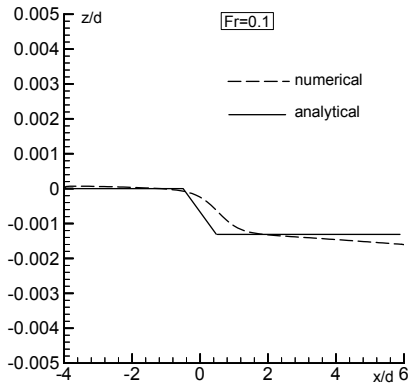
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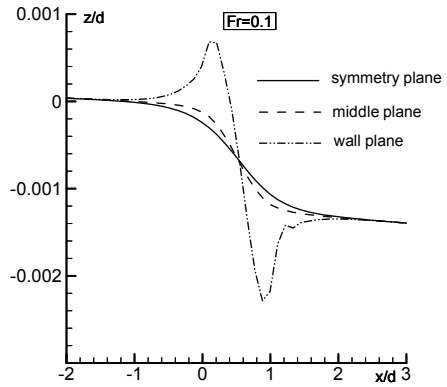
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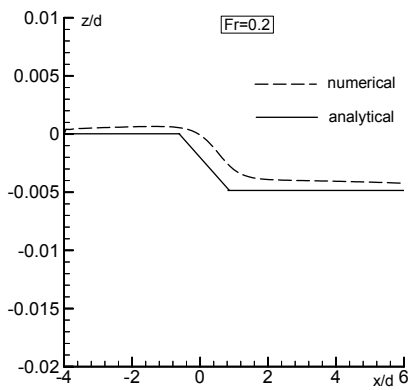


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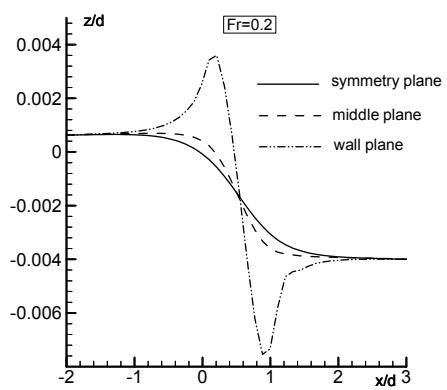


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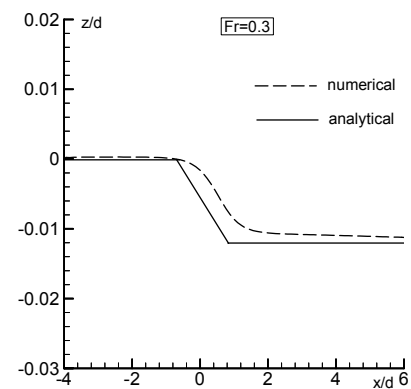


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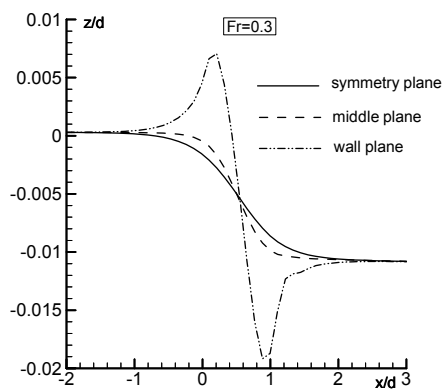


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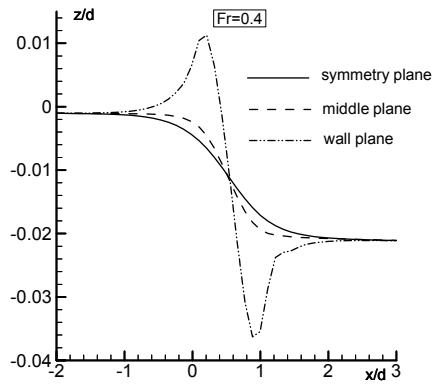
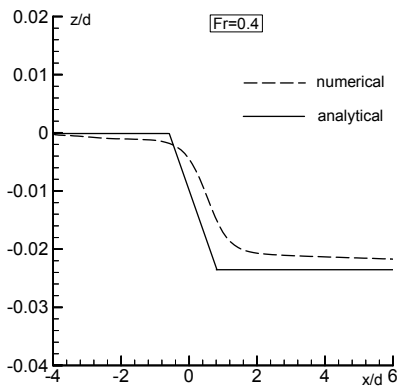
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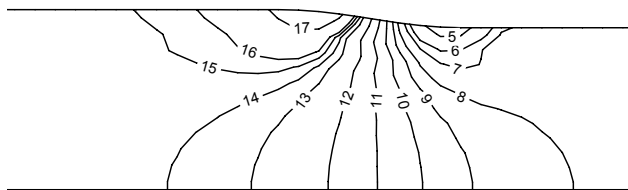
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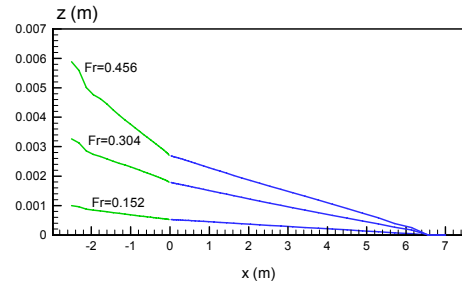
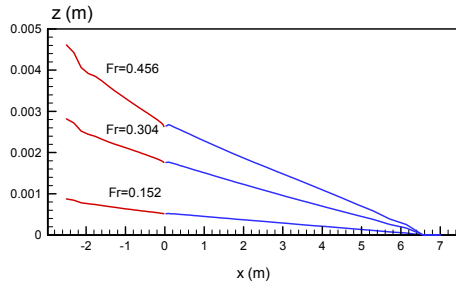
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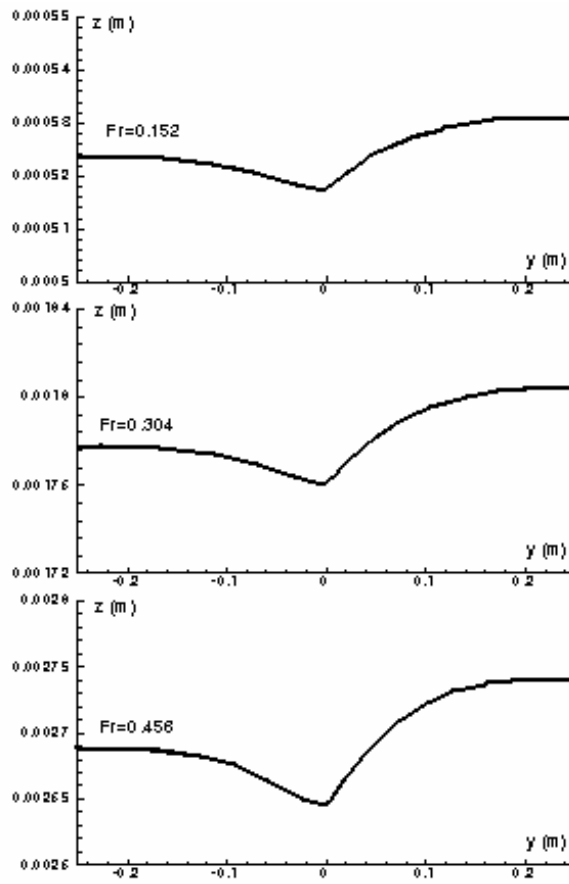
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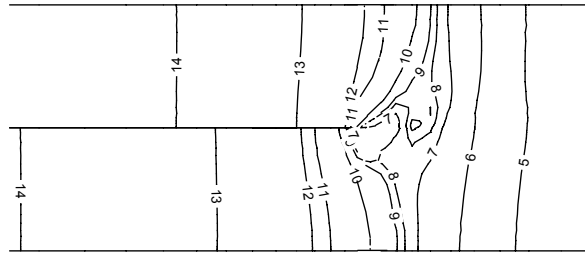


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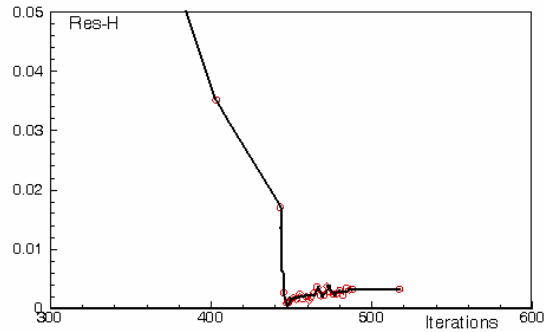
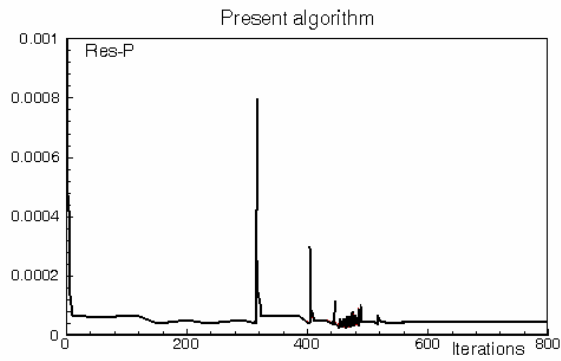


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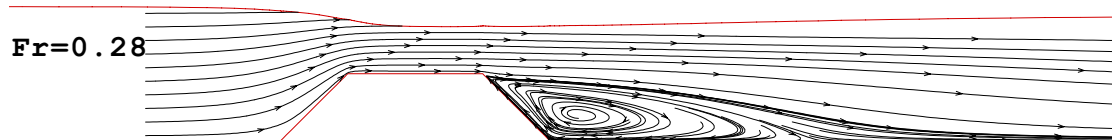
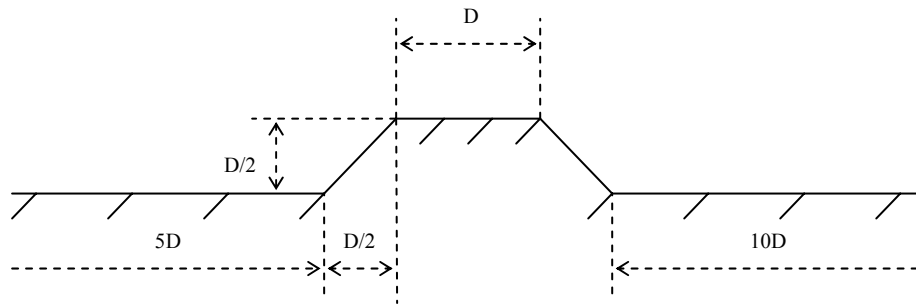
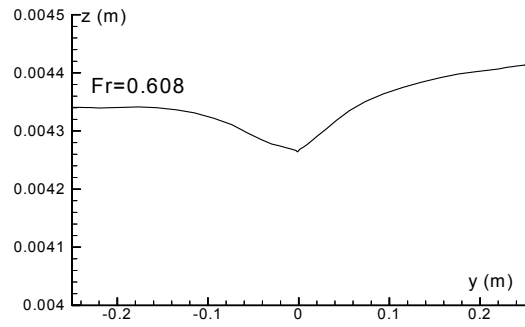
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atm

C_1, C_2, C_μ

fs

g

k

u, v, w

x, y, z

G

P

ε

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μ, μ_t

$\sigma_k, \sigma_\varepsilon$

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