

$k-\varepsilon$

$k-\varepsilon$

$k-\varepsilon$

3D Numerical Modelling of Water Circulation in River Harbours Using the Zero- and Two-Equation Turbulence Models

H. Hakimzadeh

Faculty of Civil Engineering, Sahand University of Technology

Abstract

In this paper details of the numerical model results of the layer integrated model for the square river harbour, using the zero- and two-equation turbulence models are discussed. For the zero- and two-equation turbulence models, the modified mixing length and depth integrated $k-\varepsilon$ models were deployed to calculate the horizontal eddy viscosity coefficients, respectively. Likewise, the parabolic distribution and layer integrated $k-\varepsilon$ models were used to determine the vertical eddy viscosity. The model was applied for prediction of flow within the river and harbour and the various numerical model and experimental results were then compared graphically with each other. General study of the numerical model results showed that the zero-equation model has predicted small values for the velocity components within the harbour, whereas the two-equation model has predicted reasonable values for the flow components and these values were relatively in good agreement with the experimental results. Also, the numerical model results of the vertical profile of horizontal velocity components have shown the capability of the $k-\varepsilon$ turbulence model in predicting the experimental data.

Key words: Numerical models, Three-dimensional flow, Finite difference, River harbour, Turbulence models.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad ()$$

$$\frac{\partial u}{\partial t} + \frac{\partial uu}{\partial x} + \frac{\partial uv}{\partial y} + \frac{\partial uw}{\partial z} = f_c v - \frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\partial(-\overline{u'u'})}{\partial x} + \frac{\partial(-\overline{u'v'})}{\partial y} + \frac{\partial(-\overline{u'w'})}{\partial z} \quad ()$$

[] Raithby

Lin [] Spaulding Huang [] Kobayashi Myong

[] Falconer

$$\frac{\partial u}{\partial t} + \frac{\partial vu}{\partial x} + \frac{\partial vw}{\partial y} + \frac{\partial vw}{\partial z} = -f_c u - \frac{1}{\rho} \frac{\partial p}{\partial y} + \frac{\partial(-\overline{v'u'})}{\partial x} + \frac{\partial(-\overline{v'v'})}{\partial y} + \frac{\partial(-\overline{v'w'})}{\partial z} \quad ()$$

Bijvelds [] Wai Lu []

[]

$$\frac{\partial p}{\partial z} + \rho g = 0 \quad ()$$

[] Regab Jodan []

Langendoen

[] Nece

$$= w \quad v \quad u \quad = t$$

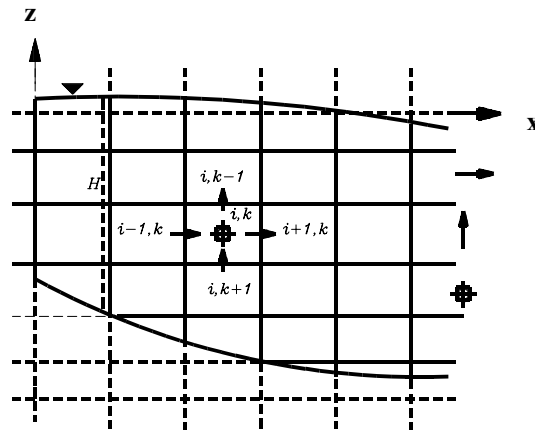
$$= \rho \quad = p \quad z \quad y \quad x$$

$$(-\overline{u'u'}) \quad = g \quad = f_c$$

()

()

() x-z



$$q_{lx} = \bar{u} \Delta z$$

$$q_{lx}, q_{ly} \quad q_{ly} = \bar{v} \Delta z$$

[]

$$\left. \frac{\partial q_{lx}}{\partial t} \right|_k + \left[\beta_l \left(\frac{\partial \bar{u} q_{lx}}{\partial x} + \frac{\partial \bar{v} q_{lx}}{\partial y} \right) \right]_k =$$

$$f_c q_{ly} \Big|_k - g \Delta z \left. \frac{\partial \zeta}{\partial x} \right|_k +$$

$$\left\{ \frac{\partial}{\partial x} v_{th} \Delta z \left[\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{u}}{\partial x} \right] + \frac{\partial}{\partial y} v_{th} \Delta z \left[\frac{\partial \bar{u}}{\partial y} + \frac{\partial \bar{v}}{\partial x} \right] \right\}_k$$

$$+ (w\bar{u})_{k+1/2} - (w\bar{u})_{k-1/2} + [(-\bar{u}'w')_{k-1/2} - (-\bar{u}'w')_{k+1/2}] \quad ()$$

$$\left. \frac{\partial q_{ly}}{\partial t} \right|_k + \left[\beta_l \left(\frac{\partial \bar{u} q_{ly}}{\partial x} + \frac{\partial \bar{v} q_{ly}}{\partial y} \right) \right]_k =$$

$$- f_c q_{lx} \Big|_k - g \Delta z \left. \frac{\partial \zeta}{\partial y} \right|_k +$$

$$\left\{ \frac{\partial}{\partial x} v_{th} \Delta z \left[\frac{\partial \bar{v}}{\partial x} + \frac{\partial \bar{u}}{\partial y} \right] + \frac{\partial}{\partial y} v_{th} \Delta z \left[\frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{v}}{\partial y} \right] \right\}_k$$

$$+ (w\bar{v})_{k+1/2} - (w\bar{v})_{k-1/2} + [(-\bar{v}'w')_{k-1/2} - (-\bar{v}'w')_{k+1/2}] \quad ()$$

$$w_{k-1/2} = - \sum_{k=k}^K \left\{ \frac{\partial(\bar{u}\Delta z)}{\partial x} + \frac{\partial(\bar{v}\Delta z)}{\partial y} \right\} \quad ()$$

$$\bar{v} \quad \bar{u} \quad = \Delta z \quad k-1/2$$

y x

[]

$$\frac{\partial \zeta}{\partial t} + \sum_{k=1}^K \left\{ \frac{\partial(\bar{u}\Delta z)}{\partial x} + \frac{\partial(\bar{v}\Delta z)}{\partial y} \right\} = 0 \quad ()$$

= ζ

$$\begin{aligned} & \frac{\partial \bar{k}H}{\partial t} + \frac{\partial \bar{k}UH}{\partial x} + \frac{\partial \bar{k}VH}{\partial y} = \\ & \frac{\partial}{\partial x} \left(\frac{\bar{v}_{th}H}{\sigma_k} \cdot \frac{\partial \bar{k}}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{\bar{v}_{th}H}{\sigma_k} \cdot \frac{\partial \bar{k}}{\partial y} \right) \\ & + \bar{v}_{th}H \left[2 \left(\frac{\partial U}{\partial x} \right)^2 + 2 \left(\frac{\partial V}{\partial y} \right)^2 + \left(\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} \right)^2 \right] \\ & + c_k U_*^3 - \bar{\epsilon}H \end{aligned} \quad ()$$

$$\beta_l = \frac{\int_{k+1/2}^{k-1/2} u^2 dz}{\bar{u}^2 \Delta z} :$$

v_{th}

v_{tv}

($k=1$)

$$\begin{aligned} & \frac{\partial \bar{\epsilon}H}{\partial t} + \frac{\partial \bar{\epsilon}UH}{\partial x} + \frac{\partial \bar{\epsilon}VH}{\partial y} = \\ & \frac{\partial}{\partial x} \left(\frac{\bar{v}_{th}H}{\sigma_\epsilon} \cdot \frac{\partial \bar{\epsilon}}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{\bar{v}_{th}H}{\sigma_\epsilon} \cdot \frac{\partial \bar{\epsilon}}{\partial y} \right) + \\ & c_{1\epsilon} c_\mu \bar{k}H \left[2 \left(\frac{\partial U}{\partial x} \right)^2 + 2 \left(\frac{\partial V}{\partial y} \right)^2 + \left(\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} \right)^2 \right] \\ & + c_\epsilon \frac{U_*^4}{H} - c_{2\epsilon} \frac{\bar{\epsilon}^2}{\bar{k}} H \end{aligned} \quad ()$$

$$\begin{aligned} & (w\bar{v})_{k-1/2} \quad (w\bar{u})_{k-1/2} \\ & (w\bar{v})_{k+1/2} \quad (w\bar{u})_{k+1/2} \end{aligned}$$

$$= \bar{\epsilon} \quad \bar{k} \quad []$$

x

[] Fischer

$$c_\epsilon = 3.6 c_{2\epsilon} c_\mu^{1/2} (f/2)^{-3/4} \quad y$$

$$= f \bar{v}_{th} = c_\mu \frac{\bar{k}^2}{\bar{\epsilon}} \quad c_k = (f/2)^{-1/2}$$

$$v_{th} = 0.15 U_* H \quad ()$$

$\sigma_k, \sigma_\epsilon, c_\mu, c_{1\epsilon}, c_{2\epsilon}$

$= H$

$= U_*$

$$\hat{k} = \frac{1}{\Delta z} \int_{k+1/2}^{k-1/2} k dz$$

[]

$k-\epsilon$

[]

$$v_{tv} = \kappa u_* (z+h) \left(1 - \frac{z+h}{H} \right) \quad ()$$

$$\left. \frac{\partial \hat{k} \Delta z}{\partial t} \right|_k + \left(\frac{\partial \hat{k} q_{lx}}{\partial x} + \frac{\partial \hat{k} q_{ly}}{\partial y} \right)_k +$$

Von Karman

$= \kappa$

$= h$

$$(w\hat{k})_{k-1/2} - (w\hat{k})_{k+1/2} =$$

$$\left(\frac{\hat{v}_{tv}}{\sigma_k} \cdot \frac{\partial \hat{k}}{\partial z} \right)_{k-1/2} - \left(\frac{\hat{v}_{tv}}{\sigma_k} \cdot \frac{\partial \hat{k}}{\partial z} \right)_{k+1/2}$$

[]

$k-\epsilon$

$$+ \hat{P} \Delta z - \hat{\epsilon} \Delta z \quad ()$$

$$\begin{aligned}
 & \frac{\partial \hat{\varepsilon} \Delta z}{\partial t} \Big|_k + \left(\frac{\partial \hat{\varepsilon} q_{lx}}{\partial x} + \frac{\partial \hat{\varepsilon} q_{ly}}{\partial y} \right) \Big|_k \\
 & + (w \hat{\varepsilon})_{k-1/2} - (w \hat{\varepsilon})_{k+1/2} = \\
 & \left(\frac{\hat{v}_{tv}}{\sigma_\varepsilon} \cdot \frac{\partial \hat{\varepsilon}}{\partial z} \right) \Big|_{k-1/2} - \left(\frac{\hat{v}_{tv}}{\sigma_\varepsilon} \cdot \frac{\partial \hat{\varepsilon}}{\partial z} \right) \Big|_{k+1/2} \\
 & + c_{1\varepsilon} \frac{\hat{\varepsilon}}{\hat{k}} \hat{P} \Delta z - c_{2\varepsilon} \frac{\hat{\varepsilon}^2}{\hat{k}} \Delta z \quad ()
 \end{aligned}$$

[]

$$k_w = \frac{u_*^2}{\sqrt{c_\mu}} \quad ()$$

$$\hat{P} = \hat{v}_{tv} \left[\left(\frac{\partial \bar{u}}{\partial z} \right)^2 + \left(\frac{\partial \bar{v}}{\partial z} \right)^2 \right] \quad ()$$

$$\varepsilon_w = \frac{u_*^3}{\kappa z_c} \quad ()$$

$$\hat{v}_{tv} = c_\mu \frac{\hat{k}^2}{\hat{\varepsilon}} \quad ()$$

= z_c

Rodi

Rodi Demuren

[]

[]

$$k_d = 0.004 u_d^2 \quad ()$$

$$\varepsilon_d = c_\mu^{3/4} \frac{k_d^{3/2}}{0.09 b} \quad ()$$

= b

= u_d

[] Rodi

$$(-\overline{u'w'}) \Big|_{-h} = v_{tv} \frac{\partial \bar{u}}{\partial z} = u_*^2 \quad ()$$

= u_{*}

= ρ(-u'w')_{-h}

[]

[]

$$(-\overline{u'w'}) \Big|_{-h} = \bar{u} (\bar{u}^2 + \bar{v}^2)^{1/2} \left[2.5 \ln \left(\frac{30d}{k_s} \right) \right]^{-2}$$

()

[]

= k_s

= d

[]

()

()

()

[]

$l \text{ cm} \times l \text{ cm}$

()

l

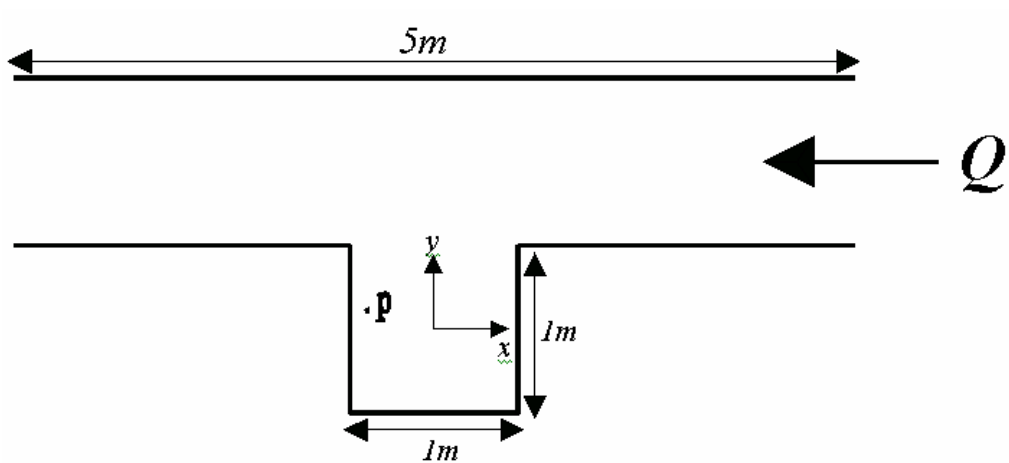
()

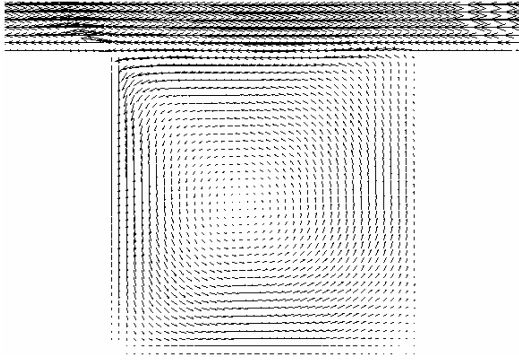
l

l

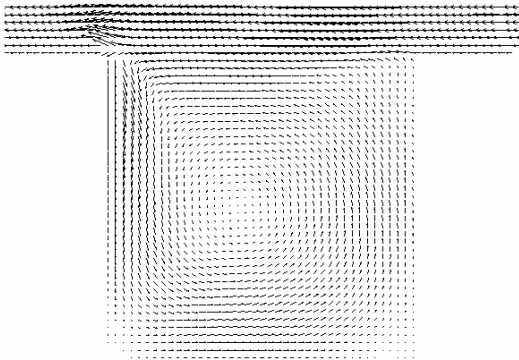
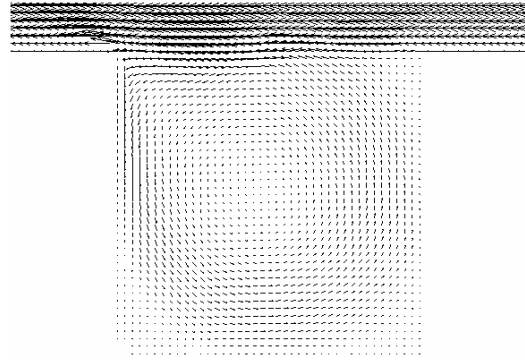
k_s

[]

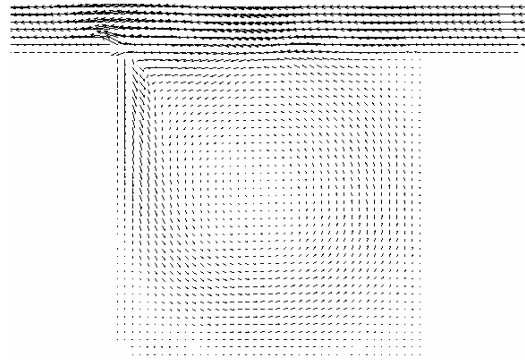




$k-\varepsilon$



$k-\varepsilon$



()

()

$k-\varepsilon$

()

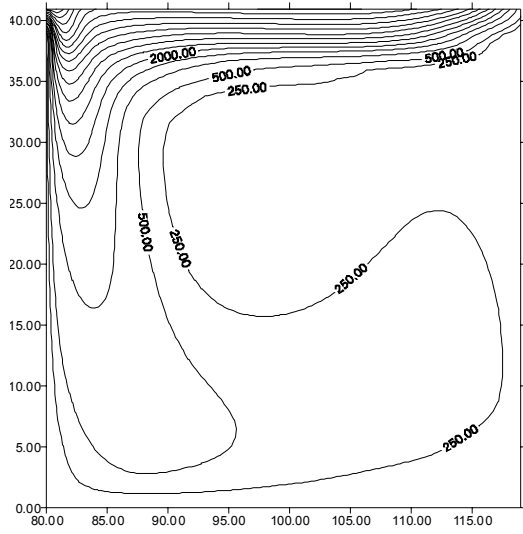
[]

()

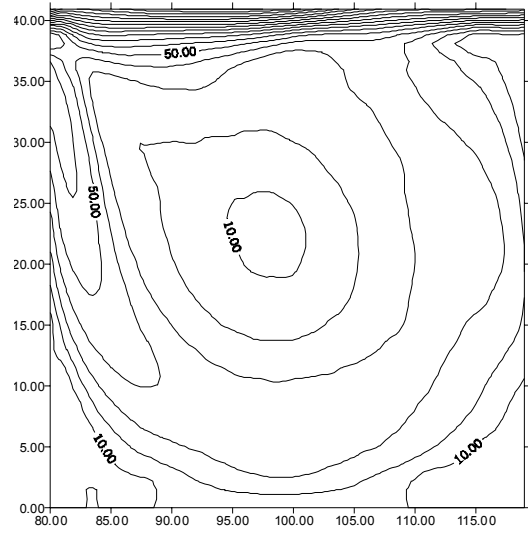
()

()

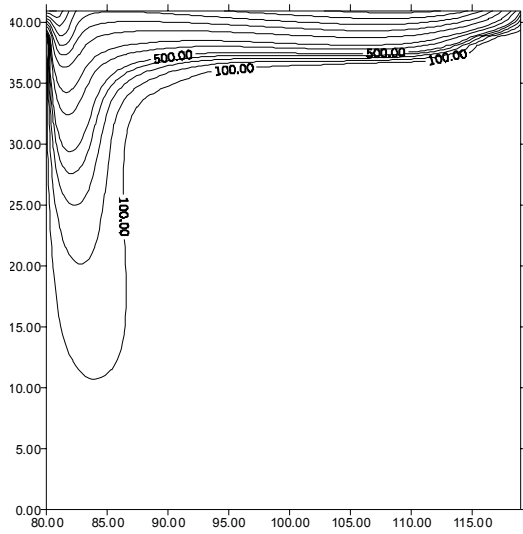
$k-\varepsilon$



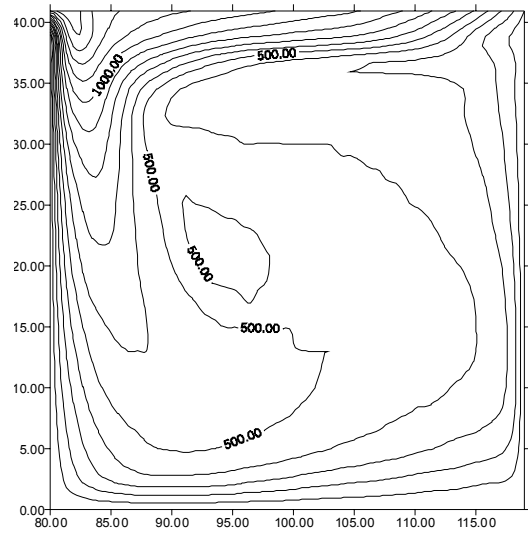
$k-\epsilon$
()



()



$k-\epsilon$
()



$k-\epsilon$
()

$k-\epsilon$

Bijvelds
()

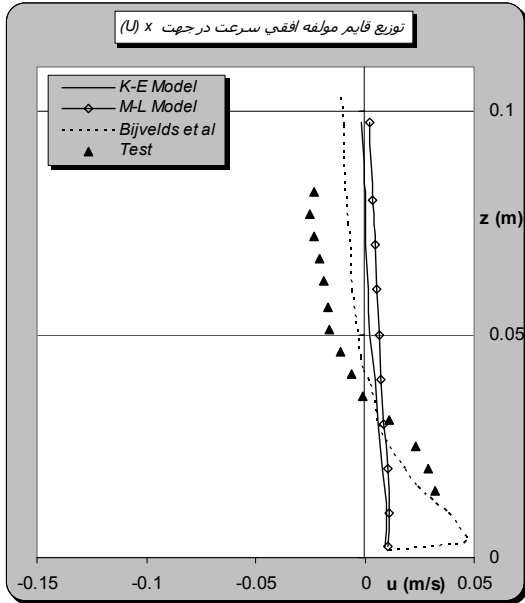
$k-\epsilon$

[]

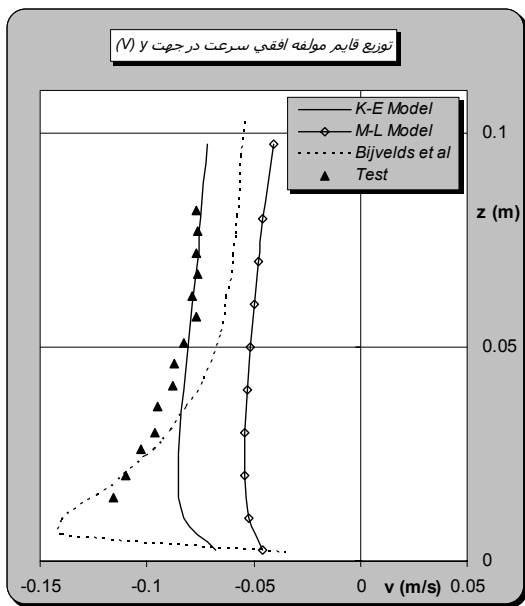
$k-\epsilon$

[]

Bijvelds



x



y

[]

()

y x

P

k-ε

Bijvelds

[]

x

Bijvelds

y

Bijvelds

k-ε

k-ε

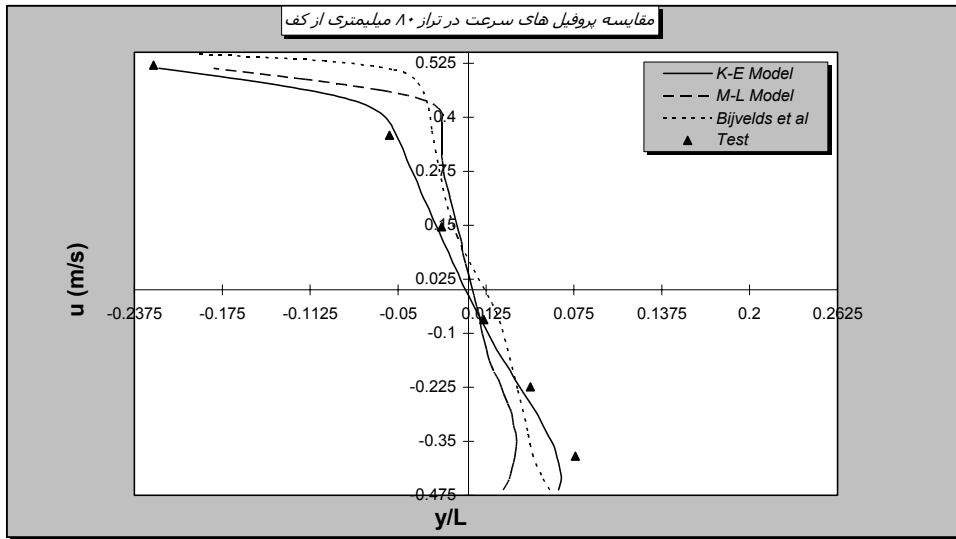
$k-\varepsilon$

Bijvelds

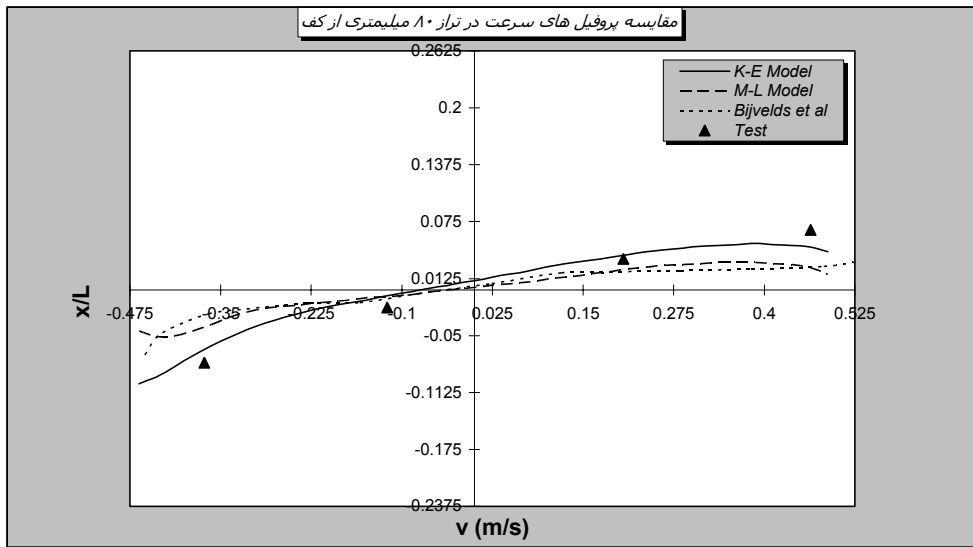
[]

y x

L



x

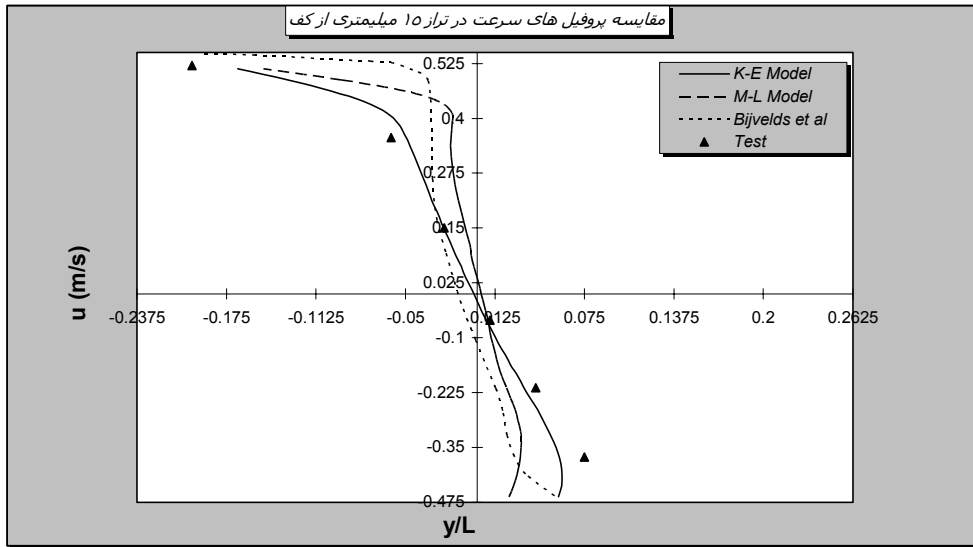


y

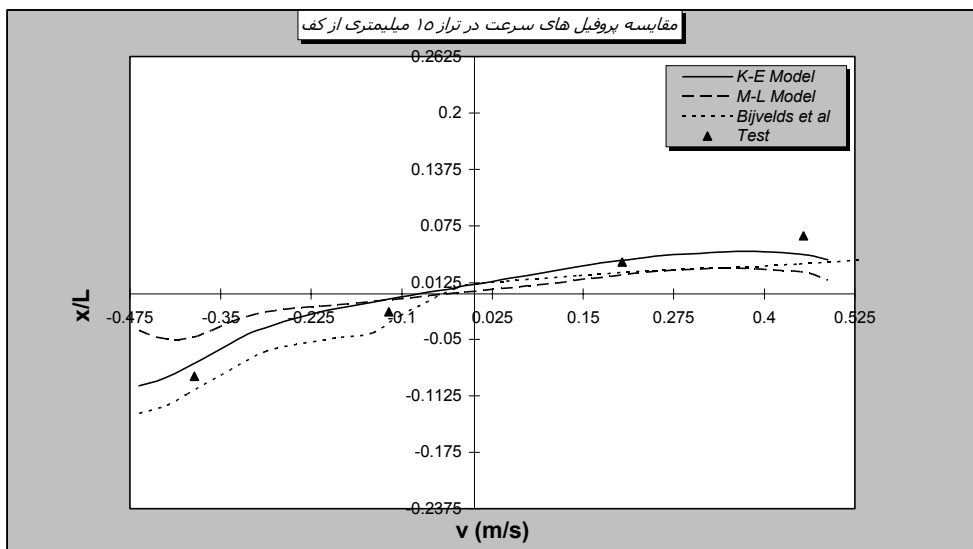
$k-\varepsilon$

$k-\varepsilon$

()



x



y

- Hydraulic Engineering, Vol. 114, No.7, pp. 720-737.
- [2] Myong, H.K., and Kobayashi, T. (1991) "Prediction of Three-Dimensional Developing Turbulent Flow in a Square Duct With an Anisotropic Low-Reynolds-Number K- ϵ Model", Journal of Fluid Engineering, Transactions of the ASME, Vol. 113, pp. 608-615. $k-\epsilon$ ()
- [3] Huang, W. and Spaulding, M. (1995) "3D Model of Estuarine Circulation and Water Quality Induced by Surface Discharges", Journal of Hydraulic Engineering, Vol. 121, No.4, pp. 300-311. $k-\epsilon$
- [4] Lin, B. and Falconer, R.A. "Three-Dimensional Layer Integrated Modelling of Estuarine Flows with Flooding and Drying", Estuarine, Coastal and Shelf Science, Academic Press Ltd (1997).
- [5] Ghiassi, R. (1995) "Three Dimensional Coastal Flow Modelling Using the Finite Volume Method", PhD Thesis, University of Bradford, Bradford, UK.
- [6] Lu, Q. and Wai, W.H. (1996) "An Efficient Splitting Method With FEM and FDM for 3-D Hydrodynamic Computations", Proceedings of Second International Conference on Hydrodynamics, Eds. A.T. Chwang, J.H.W. Lee & D.Y.C. Leung, Volume 2, pp. 697-702. $k-\epsilon$
- [7] Bijvelds, M.D.J.P., Kranenburg, C. and Stelling, G.S., (1999). "3D Numerical Simulation of Turbulent Shallow-Water Flow in Square Harbour", Journal of Hydraulic Engineering., Vol. 125, No.1, pp. 26-31. $k-\epsilon$ $k-\epsilon$
- [8] Langendoen, E.J., Booij, R., and Kranenburg, C., (1994). "Flow patterns and exchange of matter in tidal harbours", Journal of Hydraulic Research., Delft, The Netherlands, 32(2), 259-270.
- [9] Jodan, S.A. and Regab, S.A., (1994) "On the Unsteady and Turbulent Characteristics of the Three-Dimensional Shear-Driven Cavity Flow", Journal of Fluid Engineering, Transactions of the ASME, Vol. 116, No.3, pp. 439-449.
- [10] Nece, R.E., (1984) "Planform Effects on Tidal Flushing of Marinas", Journal of Waterway, Port, Coastal and Ocean Eng., ASCE, Vol. 110, No.2, pp. 251-269. $k-\epsilon$
- [11] Fischer, H.B., (1973). "Longitudinal Dispersion and Turbulent Mixing in Open Channel Flow", Annual Review of Fluid Mechanics, 5, pp. 59-78.
- [1] Raithby, G.D., Elliott, R.V. and Hutchinson, B.R. (1988) "Prediction of Three-Dimensional Thermal Discharge Flows", Journal of

-
- [17] Falconer, R.A. and Hakimzadeh, H., (1995). "Numerical modelling of secondary tide induced circulation in rectangular harbours with large aspect ratios" Proceedings of Third International Conference on Planning, Design and Operation of Marinas, ed. W.R. Blain, Computational Mechanics Publications, pp. 109-118.
- [18] Hakimzadeh, H. (2002). "Simulation of Water Circulation in River Harbour Using Zero- and Two-Equation Turbulence Models", Journal of Faculty of Engineering, University of Tabriz, Iran, No. 27, Autumn & Winter 2002, pp. 1-12. (in Persian).
- [19] Hakimzadeh, H. (2001). "Numerical Modelling of Cavity Flow in a Square Harbour Using the k- ϵ Turbulence Model", Proceedings of International Conference on Hydraulic Structures, Shahid Bahonar University of Kerman, Iran.
- [20] Langendoen, E.J., (1992). "Flow patterns and transport of dissolved matter in tidal harbours", PhD Thesis, Delft University of Technology, Delft, The Netherlands.
- [12] Rodi, W. "Elements of the Theory of Turbulence", in Coastal, Estuarial and Harbour Engineer's Reference Book, eds. Abbott and Price, E and F N Spon Ltd., London, pp. 45-59 (1993).
- [13] Falconer, R.A. and Li, G. "Numerical Modelling of Tidal Eddies in Coastal Basins with Narrow Entrance Using k- ϵ Turbulence Model", in Mixing and Transport in the Environment, eds. K.J. Beven, et. al., John Wiley & Sons Ltd., London, pp. 325-350 (1994).
- [14] Hakimzadeh, H. (1997) "Turbulence Modelling of Tidal Currents in Rectangular Harbours", PhD Thesis, University of Bradford, Bradford, UK.
- [15] Rodi, W. (1984) "Turbulence Models and their Application in Hydraulics", IAHR, Second Edition, Delft, The Netherlands, pp. 1-104.
- [16] Demuren, A. and Rodi, W. (1983) "Three-Dimensional Calculation of Side Discharge into Open Channels", Journal of Hydraulic Engineering, Vol.109, No. 12, pp. 1707-1722.