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Kelsey

Rendler []

[] Vigness

¹ Hole-Drilling Strain gage method

² Through hole

³ Blind Hole

[] Beaney Procter .

[] Schajer .

[] Flaman .

[] Schajer .

[] ASTM E837

$$\sigma_x \quad P(R, \alpha) \quad ()$$
$$\sigma'_R = \frac{\sigma_x}{2}(1 + \cos 2\alpha) \quad ()$$
$$\sigma'_\theta = \frac{\sigma_x}{2}(1 - \cos 2\alpha) \quad ()$$
$$\tau'_{r\theta} = -\frac{\sigma_x}{2}(\sin 2\alpha) \quad ()$$
$$\sigma_x \quad P(R, \alpha) \quad ()$$
$$() \quad ()$$

$$\varphi = \varphi(R, \theta) \quad ()$$

$$\varphi = f_0(R) + f_2(R) \cos 2\theta \quad ()$$

$$\nabla^4 \varphi = 0 \quad ()$$

C_1

$$f_2 \quad f_0 \quad () \quad ()$$

C_8

$$f_0(R) = C_1 R^2 \ln r + C_2 R^2 + C_3 \ln R + C_4 \quad ()$$

$$f_2(R) = C_5 R^2 + C_6 R^4 + \frac{C_7}{R^2} + C_8 \quad ()$$

$$() \quad ()$$

$$\varphi = C_1 R^2 \ln R + C_2 R^2 + C_3 \ln R + C_4 + (C_5 R^2 + C_6 R^4 + \frac{C_7}{R^2} + C_8) \cos 2\theta \quad ()$$

$$()$$

$$\sigma_R = \frac{1}{R} \frac{\partial \varphi}{\partial r} + \frac{1}{R^2} \frac{\partial^2 \varphi}{\partial \theta^2} \quad ()$$

$$\sigma_\theta = \frac{\partial^2 \varphi}{\partial R^2} \quad ()$$

:

$$\sigma_R = C_1(1 + 2 \ln R) + 2C_2 + \frac{C_3}{R^2} - \left(2C_5 + \frac{6C_7}{R^4} + \frac{4C_8}{R^2} \right) \cos 2\alpha \quad ()$$

$$\sigma_\theta = C_1(3 + 2 \ln R) + 2C_2 + \frac{C_3}{R^2} - \left(2C_5 + 12C_6 R^2 + \frac{6C_7}{R^2} \right) \cos 2\alpha \quad ()$$

σ_x

$P(R, \alpha)$

R

R_0

$$\sigma_r'' = \frac{\sigma_x}{2} \left(1 - \frac{1}{r^2} \right) + \frac{\sigma_x}{2} \left(1 + \frac{3}{r^4} - \frac{4}{r^2} \right) \cos 2\alpha \quad ()$$

$$\sigma_\theta'' = \frac{\sigma_x}{2} \left(1 + \frac{1}{r^2} \right) - \frac{\sigma_x}{2} \left(1 + \frac{3}{r^4} \right) \cos 2\alpha \quad ()$$

$$r = \frac{R}{R_0 = a} \quad (R \geq R_0)$$

σ_x

$$\Delta\sigma_r = \frac{\sigma_x}{2} \left[-\frac{1}{r^2} + \left(\frac{3}{r^4} - \frac{4}{r^2} \right) \cos 2\alpha \right] \quad ()$$

$$\Delta\sigma_\theta = \frac{\sigma_x}{2} \left[\frac{1}{r^2} - \frac{3}{r^4} \cos 2\alpha \right] \quad ()$$

() ()

 $P(R, \alpha)$

() ()

$$\varepsilon_r = -\frac{\sigma_x(1+\nu)}{2E} \left[\frac{1}{r^2} - \frac{3}{r^4} \cos 2\alpha + \frac{4}{r^2(1+\nu)} \cos 2\alpha \right] \quad ()$$

$$\varepsilon_\theta = -\frac{\sigma_x(1+\nu)}{2E} \left[-\frac{1}{r^2} + \frac{3}{r^4} \cos 2\alpha - \frac{4}{r^2(1+\nu)} \cos 2\alpha \right] \quad ()$$

:

$$\varepsilon_r = \sigma_x (A + B \cos 2\alpha) \quad ()$$

$$\varepsilon_\theta = \sigma_x (-A + C \cos 2\alpha) \quad ()$$

:

$$A = -\frac{1+\nu}{2E} \left(\frac{1}{r^2} \right) \quad ()$$

$$B = -\frac{1+\nu}{2E} \left[\left(\frac{4}{1+\nu} \right) \frac{1}{r^2} - \frac{3}{r^4} \right] \quad ()$$

$$C = -\frac{1+\nu}{2E} \left[-\left(\frac{4}{1+\nu} \right) \frac{1}{r^2} + \frac{3}{r^4} \right] \quad ()$$

 $\sigma_y \quad \sigma_x$

()

$$\varepsilon_r = A(\sigma_x + \sigma_y) + B(\sigma_x - \sigma_y) \cos 2\alpha \quad ()$$

 ε_r

()

$$\varepsilon_1 = A(\sigma_x + \sigma_y) + B(\sigma_x - \sigma_y) \cos 2\alpha \quad ()$$

$$\varepsilon_2 = A(\sigma_x + \sigma_y) + B(\sigma_x - \sigma_y) \sin 2\alpha \quad ()$$

$$\varepsilon_3 = A(\sigma_x + \sigma_y) - B(\sigma_x - \sigma_y) \cos 2\alpha \quad ()$$

() ()

$$\sigma_{x,y} = \frac{\varepsilon_1 + \varepsilon_3}{4A} \pm \frac{\sqrt{(\varepsilon_1 - \varepsilon_3)^2 + (\varepsilon_3 + \varepsilon_1 - 2\varepsilon_2)^2}}{4B} \quad ()$$

$$\tan 2\alpha = \frac{\varepsilon_1 - 2\varepsilon_2 + \varepsilon_3}{\varepsilon_2 - \varepsilon_1} \quad ()$$

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$$() \quad \sigma_y \quad \sigma_x \quad ()$$

$$\varepsilon_r = \bar{A}(\sigma_x + \sigma_y) + \bar{B}(\sigma_x - \sigma_y) \cos 2\alpha \quad ()$$

$$\bar{A} = f_A(E, \nu, D_o, Z/D) \quad ()$$

$$\bar{B} = f_B(E, \nu, D_o, Z/D) \quad ()$$

Z D D_o

ν E

()

()

()

$$\sigma_{x,y} = \frac{\varepsilon_1 + \varepsilon_3}{4A} \pm \frac{\sqrt{(\varepsilon_1 - \varepsilon_3)^2 + (\varepsilon_3 + \varepsilon_1 - 2\varepsilon_2)^2}}{4B} \quad ()$$

$$\tan 2\alpha = \frac{\varepsilon_1 - 2\varepsilon_2 + \varepsilon_3}{\varepsilon_2 - \varepsilon_1} \quad ()$$

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σ_c

()

()

$$\varepsilon_r^x(1) = \sigma_x(\bar{A} + \bar{B} \cos 2\alpha_{\alpha=0}) = \sigma_c(\bar{A} + \bar{B}) \quad ()$$

$$\varepsilon_r^x(3) = \sigma_x(\bar{A} + \bar{B} \cos 2\alpha_{\alpha=90}) = \sigma_c(\bar{A} - \bar{B}) \quad ()$$

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$$\bar{A} = \frac{\varepsilon_r^x(1) + \varepsilon_r^x(3)}{2\sigma_c} \quad ()$$

$$\bar{B} = \frac{\varepsilon_r^x(1) - \varepsilon_r^x(3)}{2\sigma_c} \quad ()$$

$\varepsilon_r^x(3) \quad \varepsilon_r^x(1)$

() ()

$$\bar{A} = \frac{(\varepsilon_r^x(1)_{after} - \varepsilon_r^x(1)_{before}) + (\varepsilon_r^x(3)_{after} - \varepsilon_r^x(3)_{before})}{2\sigma_c} \quad ()$$

$$\bar{B} = \frac{(\varepsilon_r^x(1)_{after} - \varepsilon_r^x(1)_{before}) - (\varepsilon_r^x(3)_{after} - \varepsilon_r^x(3)_{before})}{2\sigma_c} \quad ()$$

ASTM E837

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[] Manning Flaman

Schajer

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¹ Incremental Strain Method
² Average Stress Method
³ Power Series Method
⁴ Integral Method

()

z

() () () () ()

() h z

$$\varepsilon_r(z) = A(\sigma_x(z) + \sigma_y(z)) + B(\sigma_x(z) - \sigma_y(z))\cos 2\alpha(z) \quad ()$$

$$A = -\frac{1+\nu}{2E} \times a \quad a = a(z, h) \quad ()$$

$$B = -\frac{1}{2E} \times b \quad b = b(z, h) \quad ()$$

() () ()

$$\varepsilon_r(z) = \frac{1+\nu}{2E} \times a(z, h) \times (\sigma_x(z) + \sigma_y(z)) + \frac{1}{2E} \times b(z, h) \times (\sigma_x(z) - \sigma_y(z))\cos 2\alpha(z) \quad ()$$

z

: h

$$\varepsilon_r(h) = \int_0^h \varepsilon_r(z) dz \quad ()$$

$$\varepsilon_r(h) = \frac{1+\nu}{2E} \int_0^h a_h(z)(\sigma_x(z) + \sigma_y(z)) dz + \frac{1}{2E} \int_0^h b_h(z)(\sigma_x(z) - \sigma_y(z))\cos 2\alpha(z) dz \quad ()$$

α $\sigma_y(z)$ $\sigma_x(z)$

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: ()

$$\varepsilon_1(h) = \frac{1+\nu}{2E} \int_0^h a_h(z)(\sigma_x(z) + \sigma_y(z)) dz + \frac{1}{2E} \int_0^h b_h(z)(\sigma_x(z) - \sigma_y(z)) \cos 2\alpha(z) dz \quad ()$$

$$\varepsilon_2(h) = \frac{1+\nu}{2E} \int_0^h a_h(z)(\sigma_x(z) + \sigma_y(z)) dz + \frac{1}{2E} \int_0^h b_h(z)(\sigma_x(z) - \sigma_y(z)) \sin 2\alpha(z) dz \quad ()$$

$$\varepsilon_3(h) = \frac{1+\nu}{2E} \int_0^h a_h(z)(\sigma_x(z) + \sigma_y(z))dz - \frac{1}{2E} \int_0^h b_h(z)(\sigma_x(z) - \sigma_y(z))\cos 2\alpha(z)dz \quad ()$$

$$\sigma_2 \quad \sigma_1 \quad () \quad \sigma_y \quad \sigma_x \quad \tau_{13}$$

$$\sigma_x(z) + \sigma_y(z) = \sigma_1(z) + \sigma_3(z) \quad ()$$

$$(\sigma_x(z) - \sigma_y(z))\cos 2\alpha = \sigma_1(z) - \sigma_3(z) \quad ()$$

:

$$\varepsilon_1(h) = \frac{1+\nu}{2E} \int_0^h a_h(z)(\sigma_1(z) + \sigma_3(z))dz + \frac{1}{2E} \int_0^h b_h(z)(\sigma_1(z) - \sigma_3(z))dz \quad ()$$

$$\varepsilon_2(h) = \frac{1+\nu}{2E} \int_0^h a_h(z)(\sigma_x(z) + \sigma_y(z))dz - \frac{1}{E} \int_0^h b_h(z)\tau_{13} dz \quad ()$$

$$\varepsilon_3(h) = \frac{1+\nu}{2E} \int_0^h a_h(z)(\sigma_x(z) + \sigma_y(z))dz - \frac{1}{2E} \int_0^h b_h(z)(\sigma_1(z) - \sigma_3(z))dz \quad ()$$

: ()

$$\varepsilon_3(h) + \varepsilon_1(h) = \frac{1+\nu}{E} \int_0^h a_h(z)(\sigma_3(z) + \sigma_1(z))dz \quad ()$$

$$\varepsilon_3(h) - \varepsilon_1(h) = \frac{1}{E} \int_0^h b_h(z)(\sigma_3(z) - \sigma_1(z))dz \quad ()$$

$$\varepsilon_3(h) + \varepsilon_1(h) - 2\varepsilon_2(h) = \frac{2}{E} \int_0^h b_h(z)\tau_{13} dz \quad ()$$

)

: (n h

$$\varepsilon_1(h) + \varepsilon_3(h) = \frac{1+\nu}{E} \sum_0^n a_h(j)(\sigma_x(j) + \sigma_y(j)) \quad ()$$

$$\varepsilon_1(h) - \varepsilon_3(h) = \frac{1}{E} \sum_0^n b_h(j)(\sigma_x(j) - \sigma_y(j)) \quad ()$$

$$\varepsilon_3(h) + \varepsilon_1(h) - 2\varepsilon_2(h) = \frac{2}{E} \sum_0^n b_h(j)\tau_{13}(j) \quad ()$$

:

$$p = \frac{\varepsilon_3 + \varepsilon_1}{2}, q = \frac{\varepsilon_3 - \varepsilon_1}{2}, t = \frac{\varepsilon_3 + \varepsilon_1 - 2\varepsilon_2}{2} \quad ()$$

$$P = \frac{\sigma_3 + \sigma_1}{2}, Q = \frac{\sigma_3 - \sigma_1}{2}, T = \tau_{13} \quad ()$$

Q q P p

() ()

i z

: ()

$$p_i = \frac{1+\nu}{E} \sum_0^n a_{ij} P_j = \frac{1+\nu}{E} (a_{i1}P_1 + a_{i2}P_2 + a_{i3}P_3 + a_{i4}P_4 + a_{i5}P_5 + \dots) \quad 1 \leq j \leq i \leq n \quad ()$$

$$q_i = \frac{1}{E} \sum_0^n b_{ij} Q_j = \frac{1}{E} (b_{i1}Q_1 + b_{i2}Q_2 + b_{i3}Q_3 + b_{i4}Q_4 + b_{i5}Q_5 + \dots) \quad 1 \leq j \leq i \leq n \quad ()$$

$$t_i = \frac{1}{E} \sum_0^n b_{ij} T_j = \frac{1}{E} (b_{i1}T_1 + b_{i2}T_2 + b_{i3}T_3 + b_{i4}T_4 + b_{i5}T_5 + \dots) \quad 1 \leq j \leq i \leq n \quad ()$$

h . h n
 Q_i P_i
 $n=4$

$$p_i = \frac{1+\nu}{E} \sum_0^n a_{ij} P_j \quad 1 \leq j \leq i \leq 4 \quad ()$$

$$q_i = \frac{1}{E} \sum_0^n b_{ij} Q_j \quad 1 \leq j \leq i \leq 4 \quad ()$$

$$t_i = \frac{1}{E} \sum_0^n b_{ij} T_j \quad 1 \leq j \leq i \leq 4 \quad ()$$

:

$$[\bar{a}][P] = \frac{E}{1+\nu} [p] \quad ()$$

$$[\bar{b}][Q] = E [q] \quad ()$$

$$[\bar{b}][T] = E [t] \quad ()$$

τ_{13} σ_2 σ_1

$$\sigma_{j \max}, \sigma_{j \min} = P_j \pm \sqrt{Q_j^2 + T_j^2} = E \left[\frac{p_i}{\bar{a}_{i,j}(1+\nu)} + \frac{\sqrt{q_i^2 + t_i^2}}{\bar{b}_{i,j}} \right] \quad ()$$

$$\beta_j = \frac{1}{2} \tan^{-1} \left(\frac{T_j}{Q_j} \right) = \frac{1}{2} \tan^{-1} \left(\frac{t_j}{q_j} \right) \quad ()$$

z h . () () ()

b_{ij} a_{ij}

a_{ij} $()$ $: h$

$$\sigma_x(z) = \sigma_y(z) = \sigma(z) \quad ()$$

$$\sigma_x(z) - \sigma_y(z) = 0 \quad ()$$

$$\varepsilon_r(h) = \frac{1+\nu}{E} \int_0^h a_h(z) \sigma(z) dz \quad ()$$

 $)$ $\varepsilon_{ir}(h)$ $(n \quad h$ $a_{ij} \quad i$

$$\varepsilon_{ir}(h) = \frac{1+\nu}{E} \sum_0^n a_{ij} \sigma_j = \frac{1+\nu}{E} (a_{i1}\sigma_1 + a_{i2}\sigma_2 + a_{i3}\sigma_3 + \dots) \quad ()$$

 a_{ij} $:$

$$\sigma_{j \neq m} = 0 \quad ()$$

 $() \quad ()$ σ_j

$$a_{ij} = \frac{E}{1+\nu} \frac{\varepsilon_{ir}}{\sigma_{ij}} \quad 1 \leq j \leq i \quad ()$$

 $()$ b_{ij} $: h$

$$\sigma_x(z) = -\sigma_y(z) = \sigma(z) \quad ()$$

$$\sigma_x(z) + \sigma_y(z) = 0 \quad ()$$

$$\varepsilon_r(h) = \frac{1}{E} \int_0^h b_h(z) \sigma(z) \cos 2\alpha(z) dz \quad ()$$

$\alpha = 0$ b_{ij} ()
 h)
 (n

$$\varepsilon_{ir}(h) = \frac{1}{E} \sum_{j=1}^n b_{ij} \sigma_j = \frac{1}{E} (b_{i1} \sigma_1 + b_{i2} \sigma_2 + b_{i3} \sigma_3 + \dots) \quad ()$$

$\alpha = 0$ $\varepsilon_{ir}(h)$
 : ()

$$b_{ij} = E \frac{\varepsilon_{ir}}{\sigma_{ij}} \quad 1 \leq j \leq i \quad ()$$

σ_{ij}

$b_{ij} \quad a_{ij}$

$i \quad j \quad b_{ij} \quad a_{ij}$

$ii \quad i$

()

C B

A

B

C C

()

$$\sigma_x(z) = \sigma_y(z) = P \quad a_{ij}$$

$$\sigma'_x = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta = P \quad ()$$

$$\begin{array}{ccc}
 & i & j \\
 b_{ij} & & a_{ij} \\
 & () & P \\
 & & \sigma_x(z) = -\sigma_y(z) = P \\
 & \sigma'_x = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta = P \cos 2\theta & () \\
 & i & j \\
 () & & \theta = 0 \\
 & & b_{ij} \\
 & & P \cos 2\theta
 \end{array}$$

$$1 \leq j \leq 4 \quad i = 4$$

$$\begin{array}{ccc}
 b_{ij} & a_{ij} & \\
 () & & \\
 b_{ij} & a_{ij} & a_{ij}
 \end{array}$$

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:C B A

: \bar{B} \bar{A}

:b a

: b_{ij} a_{ij}

: C_{1-9}

:D

: R_0 D_0

:E

:h

:P p

:Q q

:R

() :r

:Z

: α

: σ_Y σ_X

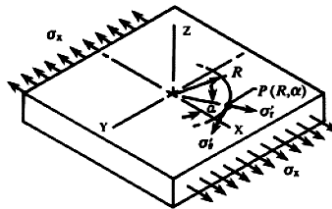
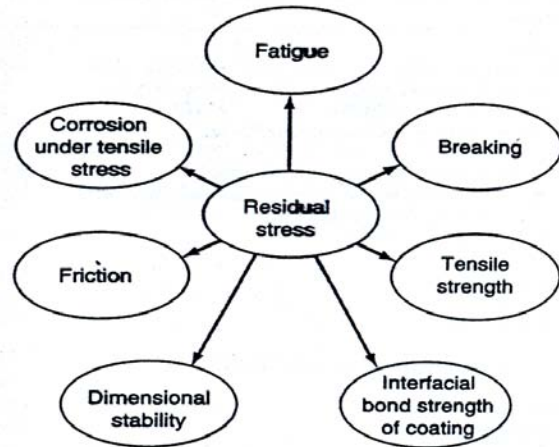
: σ'_θ σ'_r

: σ''_θ σ''_r

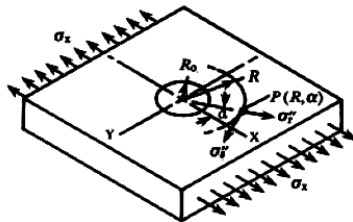
: ε_r

: φ

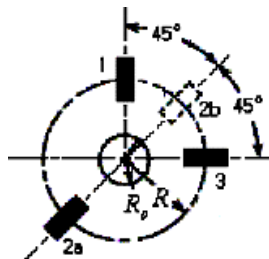
: ν

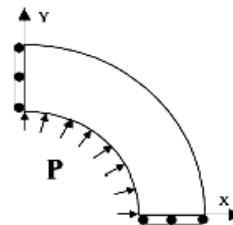
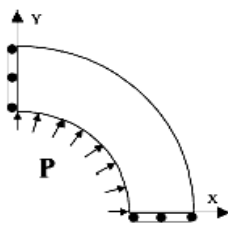
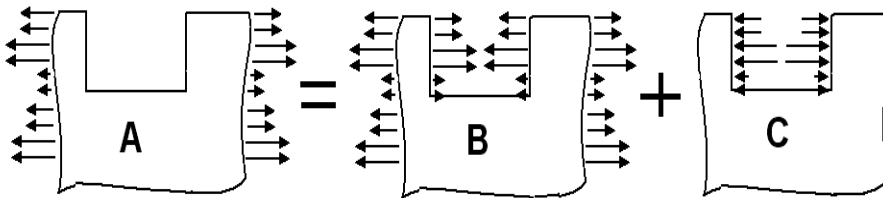
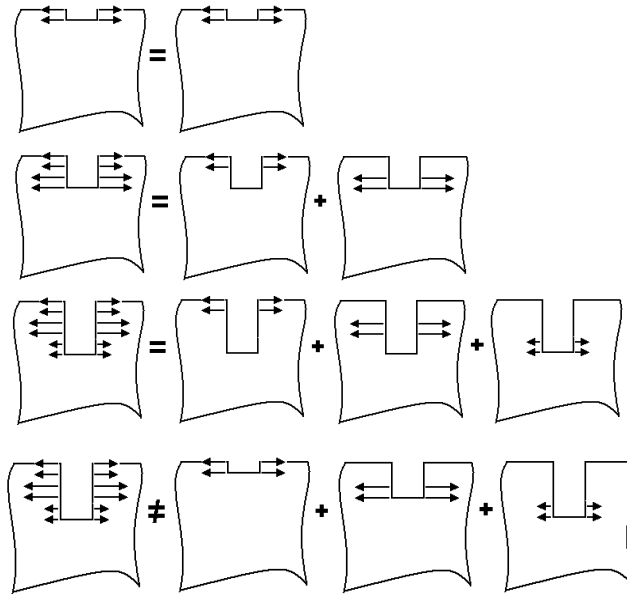
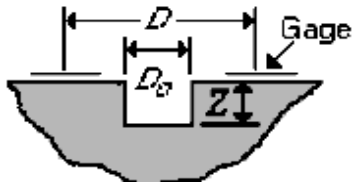


$P(R, \alpha)$



$P(R, \alpha)$

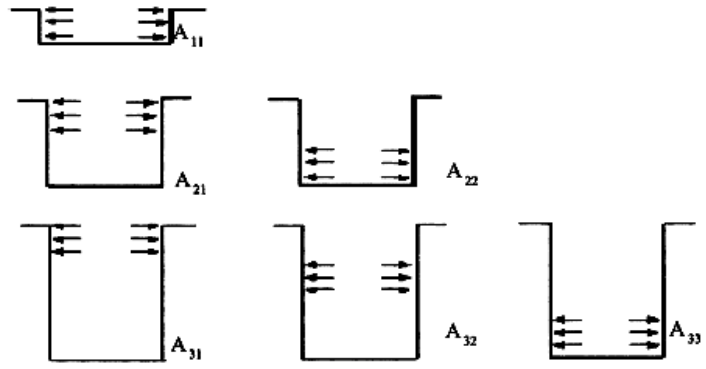




$$a_{ij} = \frac{E}{1+\nu} \frac{\varepsilon_{ir}}{P_{ij}} \quad 1 \leq j \leq i$$

$$b_{ij} = E \frac{\varepsilon_{ir}}{P_{ij}} \quad 1 \leq j \leq i$$

$$b_{ij} \quad a_{ij}$$



a_{ij}

Abstract

Non-uniform residual stresses arise from most mechanical or thermal operations, performed in processing engineering materials, like welding and machining. They may enhance occurrence of brittle fracture, fatigue, structural buckling and stress-cracking-corrosion. Therefore, estimation of their magnitude and distribution are of great importance in integrity assessments of load bearing structures. The hole-drilling strain gauge method (HDSG), described in ASTM E837, is the common method used in evaluation of through thickness uniform residual stresses. However, according to this standard, the use of this method is limited for uniform stress distribution through the thickness of the body. In this paper, the capability of the HDSG method in evaluation of non-uniform stresses is studied by using the integral technique. Here, assuming linear elastic materials, the basic relations of the method are derived. Also, a procedure for determination of the required coefficients is proposed, using by the integral method, were presented.