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AFM (Atomic Force Microscope)

(Tip)

## *Flexural Vibration of Atomic Force Microscope Cantilever with Dimensional Effects*

Anoshirvan Farshidianfar; Mohammad Hadi Mahdavi; Hamid Dalir

### **ABSTRACT**

Atomic Force microscope (AFM) is one of the powerful and useful tools in nanoscale science and technologies with applications from surface characterization in material science, to the study of living biological systems in their natural environment. AFM operate in three modes of contact, non-contact and tapping mode. In this paper, by focusing on the development of a more comprehensive model of an AFM micro-cantilever beam, considering the effects of mass and rotary inertia of the tip using Euler-Bernoulli beam theory is considered. The comparison of the present results and the results of other investigators, which has been done in case studies, generally shows a very good agreement. The results show that the effect of mass and rotary inertia of the tip depending on its dimensions is important and should be considered. Finally, the effects of cantilever inclination and tip height on the resonance frequencies are also examined.

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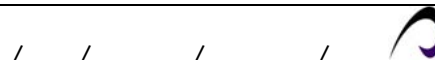
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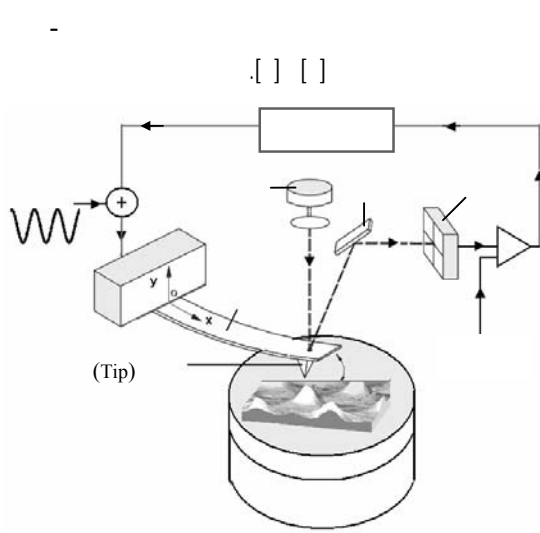
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**KEYWORDS**

Non-contact AFM – Frequency Analysis – Mass and Rotary Inertia of Tip – Euler-Bernoulli Beam.  
 MEMS



[ ]

AFM

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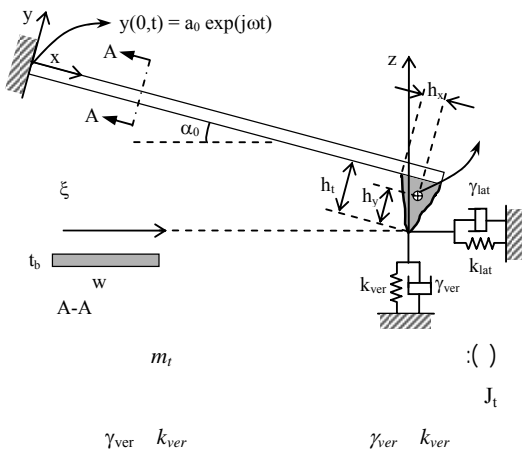
Rabe .

[ ]



$\alpha_0$   $\{\xi, z\}$  [ ] Mahdavi .

( ) [ ] :



$$\frac{\partial^4 y(x,t)}{\partial x^4} + \frac{1}{c^2} \frac{\partial^2 y(x,t)}{\partial t^2} = 0 \quad ( )$$

$$y(x,t) \quad c^2 = EI / \rho A$$

$$\omega_n = (\beta_n L)^2 \sqrt{\frac{EI}{\rho A L^4}} \quad ( ) \quad \omega \quad \beta^2 = \omega / c$$

$n=(1,2,\dots)$

$$y(0,t) = A_0 \exp(j\omega t) \quad (x=0)$$

$$L \quad A = t_b w \quad \rho \quad I = w t_b^3 / 12$$

$$y(0,t) = a_0 \exp(j\omega t) \quad ( )$$

$$\frac{\partial y}{\partial x}(0,t) = 0 \quad ( )$$

$$EI \frac{\partial^2 y(L,t)}{\partial x^2} = -EI \frac{\partial^3 y(L,t)}{\partial x^3} h_x - J_t \frac{\partial^3 y(L,t)}{\partial t^2 \partial x} + F_z \sin \alpha_0 h_y - F_\xi \cos \alpha_0 h_y \quad ( )$$

$$EI \frac{\partial^3 y(L,t)}{\partial x^3} = F_\xi \sin \alpha_0 + F_z \cos \alpha_0 + m_t \frac{\partial^2 y(L,t)}{\partial t^2} \quad ( )$$

$m_t$   $\alpha_0$   $J_t$   $h_x$   $h_y$   $h_t$   $\{x, y\}$

$$\begin{aligned}
H(\Omega) = & \Omega^2 \left[ 1 + \cos \sqrt{\Omega} \cosh \sqrt{\Omega} \right] \\
& + \Omega^{\frac{3}{2}} P_2(\Omega) \left[ \sin \sqrt{\Omega} \cosh \sqrt{\Omega} + \sinh \sqrt{\Omega} \cos \sqrt{\Omega} \right] \\
& + \Omega \left[ P_1(\Omega) + S_2(\Omega) \right] \sin \sqrt{\Omega} \sinh \sqrt{\Omega} \\
& + \sqrt{\Omega} S_3(\Omega) \left[ \sin \sqrt{\Omega} \cosh \sqrt{\Omega} - \sinh \sqrt{\Omega} \cos \sqrt{\Omega} \right] \\
& + \left[ P_2(\Omega) S_3(\Omega) - P_1(\Omega) S_2(\Omega) \right] \left[ 1 - \cos \sqrt{\Omega} \cosh \sqrt{\Omega} \right]
\end{aligned} \quad ( )$$

$$F_z = z k_{\text{ver}} + \frac{dz}{dt} \gamma_{\text{ver}} \quad ( )$$

$$F_\xi = \xi k_{\text{lat}} + \frac{d\xi}{dt} \gamma_{\text{lat}} \quad ( )$$

z     $\xi$

$$y(x, t) = y(x) \exp(j\omega t) \quad ( )$$

$$( ) \quad (\gamma_{\text{ver}} = \gamma_{\text{lat}} = k_{\text{lat}} = m_t = J_t = \alpha_0 = 0)$$

$$\begin{aligned}
H(\beta) = & \Omega^{\frac{3}{2}} \left( 1 + \cos \sqrt{\Omega} \cosh \sqrt{\Omega} \right) \\
& + K_{\text{ver}} \left( \sin \sqrt{\Omega} \cosh \sqrt{\Omega} - \cos \sqrt{\Omega} \sinh \sqrt{\Omega} \right)
\end{aligned} \quad ( )$$

[ ] Wiehn Turner

$$X = x/L, \quad Y = y/L, \quad A_0 = a_0/L$$

$$H_x = h_x/L, \quad H_y = h_y/L, \quad H_t = h_t/L$$

$$M = m_t / \rho A L, \quad J = J_t / \rho A L^3, \quad \Omega^2 = \rho A \omega^2 L^4 / EI$$

$$\Lambda_{\text{ver}} = L^3 (k_{\text{ver}} + j\omega \gamma_{\text{ver}}) / EI, \quad \Lambda_{\text{lat}} = L^3 (k_{\text{lat}} + j\omega \gamma_{\text{lat}}) / EI$$

$$: \quad ( ) \quad ( )$$

$$\frac{d^4 Y}{dX^4} - \Omega^2 Y = 0 \quad ( )$$

$$: \quad (( ) \quad ( ) \quad ( )$$

$$Y(0) = A_0 \quad ( )$$

$$\frac{dY(0)}{dX} = 0 \quad ( )$$

$$\frac{d^2 Y(1)}{dX^2} = - \left( P_1(\Omega) Y(1) + P_2(\Omega) \frac{dY(1)}{dX} \right) \quad ( )$$

$$\frac{d^3 Y(1)}{dX^3} = \left( S_3(\Omega) Y(1) + S_2(\Omega) \frac{dY(1)}{dX} \right) \quad ( )$$

( $m_{\text{tip}}/m_{\text{beam}}$ )

( / % )

$J_t$      $m_t$

( / % )

M

( )

AFM

$$P_1(\Omega) = H_x S_3(\Omega) + \frac{H_y}{H_t} S_2(\Omega) \quad ( )$$

$$P_2(\Omega) = S_1(\Omega) + H_x S_2(\Omega) \quad ( )$$

$$S_1(\Omega) = H_t H_y (\Lambda_{\text{ver}} \sin^2 \alpha_0 + \Lambda_{\text{lat}} \cos^2 \alpha_0 - \frac{\Omega^2 J}{H_t H_y}) \quad ( )$$

$$S_2(\Omega) = H_t (\Lambda_{\text{lat}} - \Lambda_{\text{ver}}) \sin \alpha_0 \cos \alpha_0 \quad ( )$$

$$S_3(\Omega) = \Lambda_{\text{ver}} \cos^2 \alpha_0 + \Lambda_{\text{lat}} \sin^2 \alpha_0 - M \Omega^2 \quad ( )$$

$$( ) \quad ( ) \quad ( )$$

$$: \quad ( )$$

$$\begin{aligned}
Y(1, \Omega) = & \frac{A_0 \Omega}{H(\Omega)} \left[ \Omega (\cos \sqrt{\Omega} + \cosh \sqrt{\Omega}) \right. \\
& + \sqrt{\Omega} P_2(\Omega) (\sin \sqrt{\Omega} + \sinh \sqrt{\Omega}) \\
& \left. + S_2(\Omega) (\cosh \sqrt{\Omega} - \cos \sqrt{\Omega}) \right]
\end{aligned} \quad ( )$$

$$k_{\text{lat}} = 0.85 k_{\text{ver}}$$

$$[ ] \quad [ ] \quad \alpha_0 = 15^\circ$$

$$(k_{\text{ver}} = \gamma_{\text{ver}} = \gamma_{\text{lat}} = k_{\text{lat}} = 0)$$

$$( ) \quad [ ]$$

$m_t$

( $\omega/\omega_{n,1}$ )

$J_t$



( )  
 ( ) ( )  
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$A_0$  ( )

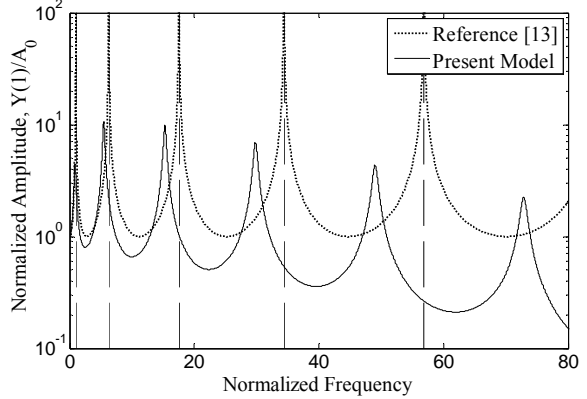
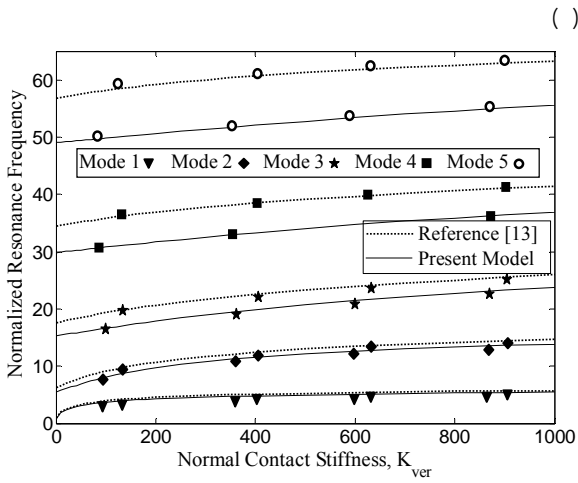
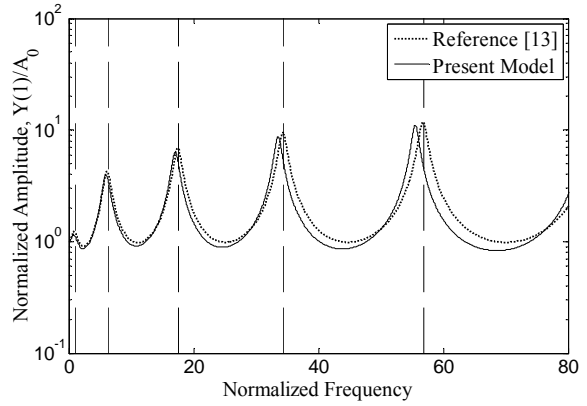
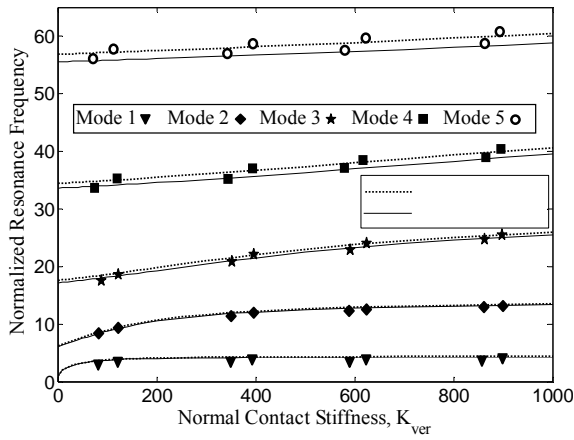
$J_t$   $m_t$

[ ] AFM : ( )

$\frac{m_{ip}}{m_{beam}}$	$h_t$ ( $\mu\text{m}$ )	$E$ (GPa)	$t_b$ ( $\mu\text{m}$ )	$w$ ( $\mu\text{m}$ )	$L$ ( $\mu\text{m}$ )
/ %	/		/		
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[ ]

[ ]



( $k_{ver} = \gamma_{ver} = \gamma_{lat} = k_{lat} = 0$ ) : ( )

( ) ( )

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[ ]

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$$J_t = 0 \quad (1)$$

$$J_t \neq 0 \quad m_t = 0$$

$$m_t \quad J_t \quad m_{tip}/m_{beam}=0 \quad [13]$$

$$J_t \quad [13] \text{ Rabe}$$

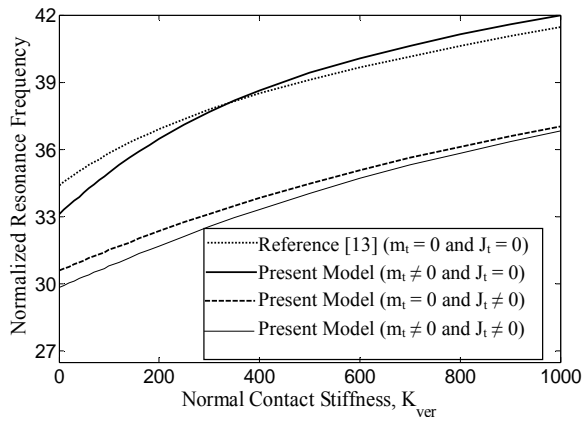
$$(\alpha_0) \quad m_t \quad / \quad \% \quad \% \quad (H_t) \quad (2)$$

$$(\alpha_0) \quad (3)$$

$$\alpha_0$$

$$\alpha_0$$

$$\alpha_0$$



$$J_t \quad m_t \quad (4)$$

$$(5)$$

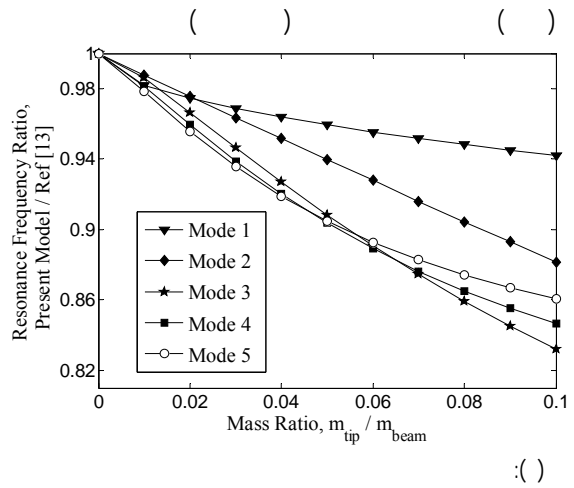
$$(h_t/L)$$

$$H_t \quad (6)$$

$$(7)$$

$$h_t/L$$

$$h_t/L \quad m_t \neq 0$$



$$(8)$$

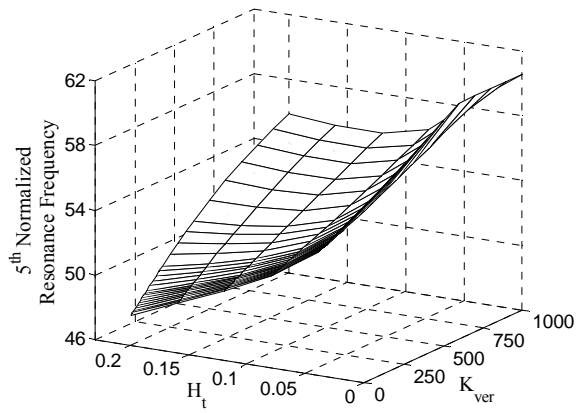
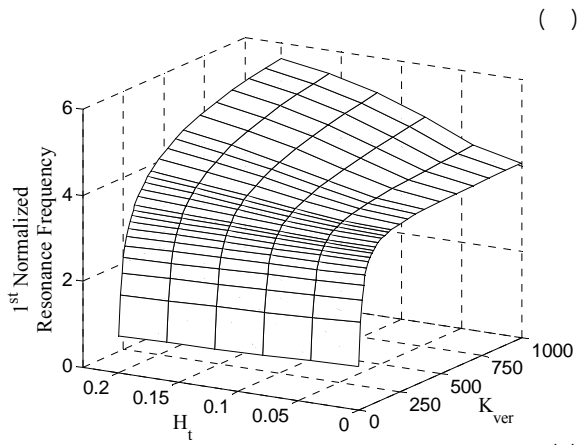
$$[13] \text{ Chang}$$

$$J_t \quad m_t$$

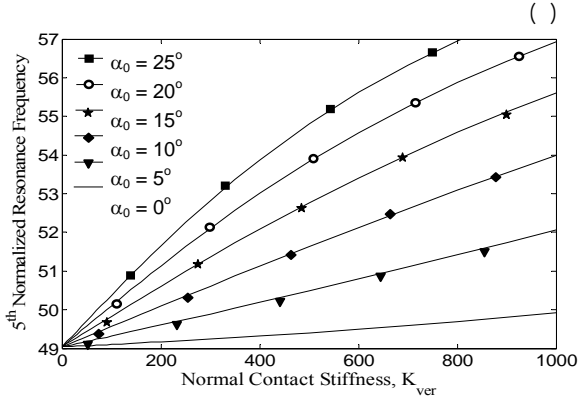
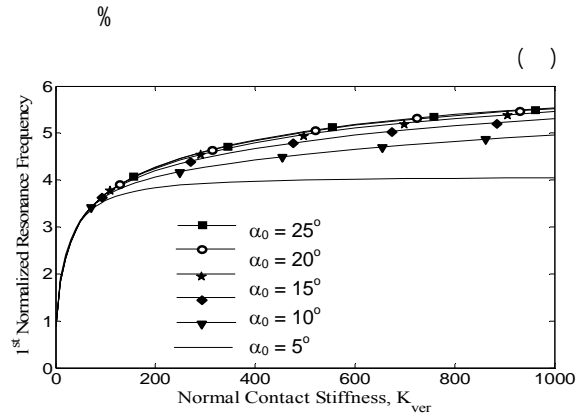


[ ] Chang

( $\alpha_0$ )

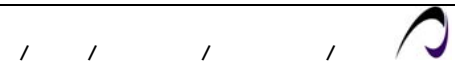


( ) ( $H_t$ ) : ( )



( ) ( $\alpha_0$ ) : ( )

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Resolution  
 Scanning Probe Microscope  
 Photo detector  
 Contact Mode  
 Non-contact Mode  
 Tapping Mode  
 Monitoring  
 Interaction  
 Lumped Model  
 Distributed Model  
 Soft  
 Stiff

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