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# *Analysis of Functionally Graded Cylindrical Vessels under Mechanical and Thermal Loads*

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### **ABSTRACT**

In this paper, a static analysis of functionally graded cylinders under axisymmetric mechanical and thermal loads is presented. It is assumed that the distribution of material properties through the thickness of the cylinder is continuous and graded according to a power law distribution. For solving equations, two-dimensional finite element method is employed. To this end, a functionally graded material element is defined for accurate finite element modeling of cylinder with continuous distribution of material properties and avoiding the limitation in the radius to thickness ratio. Numerical results are obtained for a clamped cylinder subjected to a uniform internal pressure and a simply supported cylinder subjected to a temperature distribution through the thickness. In addition, the numerical results for thick and thin cylinders are obtained. The results show that the stress and temperature distribution in functionally graded cylindrical shells are dependent on the material kind and distribution of material properties and this dependency can be utilized for controlling the stress level.

### **KEYWORDS**

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Functionally graded materials, Cylindrical vessels, Finite element method, Mechanical loading,  
Thermal loading

[ ] Jabbari

[ ] Sivakuman Awaji . (Functionally Graded Materials)

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[ ] Kitipornchai Liew .

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Shakeri . FGM

[ ] Hoseini [ ]

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[ ] Kanayama Obata .

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FGM [ ] Chin Reddy

[ ] [ ] Ozturk Tutuncu

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[ ] Noda Obata

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[ ] Horgan Chan .

[ ] Chen Ye . FGM

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Jabbari .

[ ] Meguid Woo

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$$\gamma_{r\theta} = \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}, \quad \gamma_{rz} = \frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z}, \quad \gamma_{\theta z} = \frac{\partial u_\theta}{\partial z}$$

$$\begin{Bmatrix} \sigma_z \\ \sigma_\theta \\ \sigma_r \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{12} \\ C_{12} & C_{11} & C_{12} \\ C_{12} & C_{12} & C_{11} \end{bmatrix} \begin{Bmatrix} \varepsilon_z \\ \varepsilon_\theta \\ \varepsilon_r \end{Bmatrix} - \begin{Bmatrix} \alpha \\ \alpha \\ \alpha \end{Bmatrix} \Delta T \quad (1) \quad \begin{matrix} \text{(FSDT)} \\ \text{FGM} \end{matrix}$$

$$\sigma_{r\theta} = C_{66} \gamma_{r\theta}, \quad \sigma_{rz} = C_{66} \gamma_{rz}, \quad \sigma_{z\theta} = C_{66} \gamma_{z\theta}$$

$$C_{11} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}$$

$$C_{12} = \frac{E\nu}{(1+\nu)(1-2\nu)} \quad (2) \quad \text{FGM}$$

$$C_{66} = (C_{11} - C_{12})/2$$

$$\begin{matrix} u_\theta \\ z \\ r \end{matrix} \quad (3) \quad (4)$$

$$\sigma_z = C_{11} \frac{\partial u_z}{\partial z} + C_{12} \left( \frac{u_r}{r} + \frac{\partial u_r}{\partial r} \right) - \sigma_T$$

$$\sigma_\theta = C_{11} \frac{u_r}{r} + C_{12} \left( \frac{\partial u_z}{\partial z} + \frac{\partial u_r}{\partial r} \right) - \sigma_T$$

$$\sigma_r = C_{11} \frac{\partial u_r}{\partial r} + C_{12} \left( \frac{u_r}{r} + \frac{\partial u_z}{\partial z} \right) - \sigma_T$$

$$\sigma_{r\theta} = C_{66} \left( \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r} \right) \quad (5) \quad \text{FGM}$$

$$\sigma_{rz} = C_{66} \left( \frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z} \right)$$

$$\sigma_{\theta z} = C_{66} \frac{\partial u_\theta}{\partial z}$$

$$\sigma_T = \alpha \Delta T (C_{11} + 2C_{12}) \quad (6) \quad (7)$$

$$\begin{matrix} (8) \\ (9) \end{matrix}$$

$$V = (V_{out} - V_{in}) \left( \frac{r - r_{in}}{r_{out} - r_{in}} \right)^n + V_{in} \quad (10)$$

$$r \quad V \quad V_{in} \quad V_{out}$$

$$\begin{matrix} n \\ n= \\ \text{FGM} \end{matrix}$$

$$\frac{1}{r} \frac{\partial (r\sigma_r)}{\partial r} - \frac{\sigma_\theta}{r} + \frac{\partial \sigma_{rz}}{\partial z} = \rho \frac{\partial^2 u_r}{\partial t^2} \quad (11)$$

$$\frac{1}{r} \frac{\partial (r\sigma_{rz})}{\partial r} + \frac{\partial \sigma_z}{\partial z} = \rho \frac{\partial^2 u_z}{\partial t^2}$$

$$\varepsilon_r = \frac{\partial u_r}{\partial r}, \quad \varepsilon_\theta = \frac{u_r}{r}, \quad \varepsilon_z = \frac{\partial u_z}{\partial z} \quad (12)$$



$$k_{11} = \int \left\{ C_{11} r \frac{\partial \psi_i}{\partial r} \frac{\partial \psi_j}{\partial r} + C_{12} \frac{\partial \psi_i}{\partial r} \psi_j + \frac{C_{11}}{r} \psi_i \psi_j + C_{12} \frac{\partial \psi_j}{\partial r} \psi_i + C_{66} r \frac{\partial \psi_i}{\partial z} \frac{\partial \psi_j}{\partial z} \right\} dr dz$$

$$k_{12} = k_{21} = \int \left\{ C_{12} r \frac{\partial \psi_i}{\partial r} \frac{\psi_j}{\partial z} + C_{12} \psi_i \frac{\partial \psi_j}{\partial z} + C_{66} r \frac{\partial \psi_i}{\partial z} \frac{\partial \psi_j}{\partial r} \right\} dr dz$$

$$k_{22} = \int \left\{ C_{66} r \frac{\partial \psi_i}{\partial r} \frac{\partial \psi_j}{\partial r} + C_{11} r \frac{\partial \psi_i}{\partial z} \frac{\partial \psi_j}{\partial z} \right\} dr dz$$

$$F_1 = \int \psi_i t_r ds, \quad F_2 = \int \psi_i t_z ds$$

$$Q_1 = \int \left( r \frac{\partial \psi_i}{\partial r} + \psi_i \right) \sigma_r dr dz, \quad Q_2 = \int \frac{\partial \psi_i}{\partial z} \sigma_r r dr dz \quad ( )$$

$$( ) \quad \cdot \quad z \quad r$$

$$\cdot \quad ( )$$

$$\psi_1 = \left(1 - \frac{r}{a}\right) \left(1 - \frac{z}{b}\right), \quad \psi_2 = \frac{r}{a} \left(1 - \frac{z}{b}\right)$$

$$\psi_3 = \frac{z}{b} \left(1 - \frac{r}{a}\right), \quad \psi_4 = \frac{z}{b} \frac{r}{a}$$

\*

r C

r

z

z

r

r

$$\begin{aligned} & \frac{1}{r} \frac{\partial}{\partial r} \left( C_{11} r \frac{\partial u_r}{\partial r} + C_{12} \left( u_r + r \frac{\partial u_z}{\partial z} \right) - r \sigma_r \right) - C_{11} \frac{u_r}{r^2} \\ & + \frac{\partial}{\partial z} \left( C_{66} r \left( \frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z} \right) \right) - \frac{C_{12}}{r} \left( \frac{\partial u_r}{\partial r} + \frac{\partial u_z}{\partial z} \right) + \frac{\sigma_r}{r} = 0 \\ & \frac{\partial}{\partial z} \left( C_{11} \frac{\partial u_z}{\partial z} + C_{12} \left( \frac{u_r}{r} + \frac{\partial u_r}{\partial r} \right) - \sigma_r \right) \\ & + \frac{1}{r} \frac{\partial}{\partial r} \left( C_{66} r \left( \frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z} \right) \right) = 0 \end{aligned} \quad ( )$$

$$\cdot \quad ( ) \quad \text{(weak form)}$$

$$\int \left\{ \frac{\partial w_1}{\partial r} \left( C_{11} r \frac{\partial u_r}{\partial r} + C_{12} \left( u_r + r \frac{\partial u_z}{\partial z} \right) - r \sigma_r \right) + \frac{\partial w_1}{\partial z} \left( C_{66} r \left( \frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z} \right) \right) + w_1 \left( C_{11} \frac{u_r}{r} + C_{12} \left( \frac{\partial u_z}{\partial z} + \frac{\partial u_r}{\partial r} \right) - \sigma_r \right) \right\} dr dz - \int w_1 t_r ds = 0$$

$$\int \left\{ \frac{\partial w_2}{\partial z} \left( C_{11} r \frac{\partial u_z}{\partial z} + C_{12} \left( u_r + r \frac{\partial u_r}{\partial r} \right) - r \sigma_r \right) + \frac{\partial w_2}{\partial r} \left( C_{66} r \left( \frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z} \right) \right) \right\} dr dz - \int w_2 t_z ds = 0 \quad ( )$$

$$t_r = r \sigma_r n_r + \sigma_{rz} n_z \quad ( )$$

$$t_z = \sigma_z n_z + r \sigma_{rz} n_r$$

$$\cdot \quad \psi$$

$$u_r = \sum_{j=1}^{n_z} u_j \psi_j \quad ( )$$

$$u_z = \sum_{j=1}^n v_j \psi_j$$

$$w_1 = w_2 = \psi_i$$

$$\cdot \quad ( ) \quad ( )$$

$$\int \left\{ C_{11} r \frac{\partial \psi_i}{\partial r} \frac{\partial \psi_j}{\partial r} u_j + C_{12} \frac{\partial \psi_i}{\partial r} \psi_j u_j + C_{12} r \frac{\partial \psi_i}{\partial r} \frac{\partial \psi_j}{\partial z} v_j + \frac{C_{11}}{r} \psi_i \psi_j u_j + C_{66} r \frac{\partial \psi_i}{\partial z} \frac{\partial \psi_j}{\partial r} v_j + C_{66} r \frac{\partial \psi_i}{\partial z} \frac{\psi_j}{\partial z} u_j + C_{12} \frac{\partial \psi_j}{\partial z} \psi_i v_j + C_{12} \frac{\partial \psi_j}{\partial r} \psi_i u_j - \left( r \frac{\partial \psi_i}{\partial r} + \psi_i \right) \sigma_r \right\} dr dz = \int \psi_i t_r$$

$$\int \left\{ C_{66} r \frac{\partial \psi_i}{\partial r} \frac{\partial \psi_j}{\partial r} v_j + C_{66} r \frac{\partial \psi_i}{\partial r} \frac{\partial \psi_j}{\partial z} u_j + C_{11} r \frac{\partial \psi_i}{\partial z} \frac{\psi_j}{\partial z} v_j + C_{12} \frac{\partial \psi_i}{\partial z} \psi_j u_j + C_{12} r \frac{\partial \psi_i}{\partial z} \frac{\partial \psi_j}{\partial r} u_j - \frac{\partial \psi_i}{\partial z} \sigma_r \right\} dr dz = \int \psi_i t_z ds \quad ( )$$

$$\cdot \quad ( ) \quad ( )$$

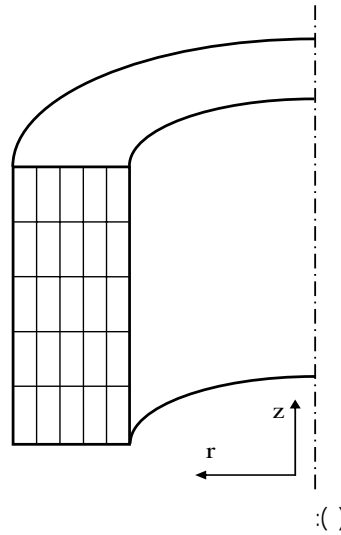
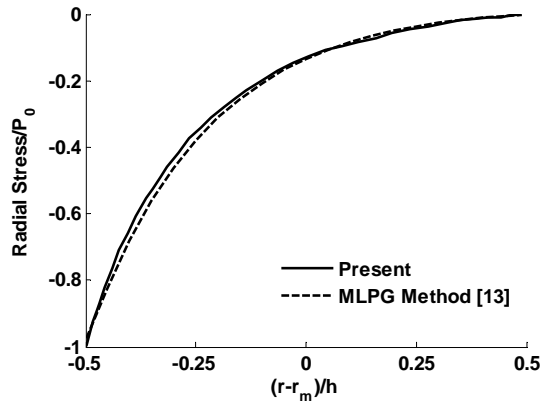
$$\begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{Bmatrix} u \\ v \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} + \begin{Bmatrix} Q_1 \\ Q_2 \end{Bmatrix} \quad ( )$$



Petrov-Galerkin

FGM

( )  $n=$



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( )

( )

$$-\frac{d}{dr} \left( k_t(r) \frac{dT}{dr} \right) = 0 \quad ( )$$

(FSDT)  $n= l$   $n$   $n$  ( )

FGM

$n= l$

$n$

) Love-Timoshenko

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(  $+z/R$  mm mm

GPa

GPa

GPa

/

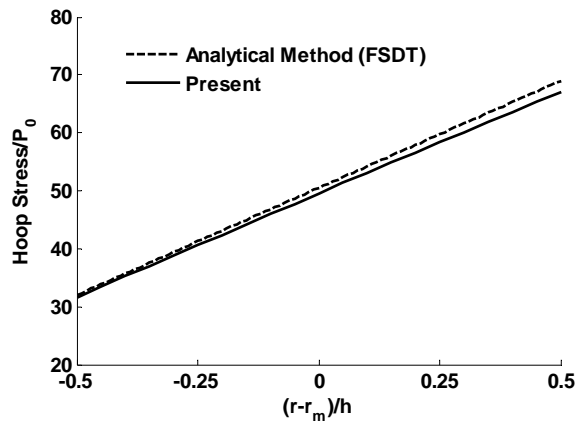
W/mK

$P_0 =$  MPa

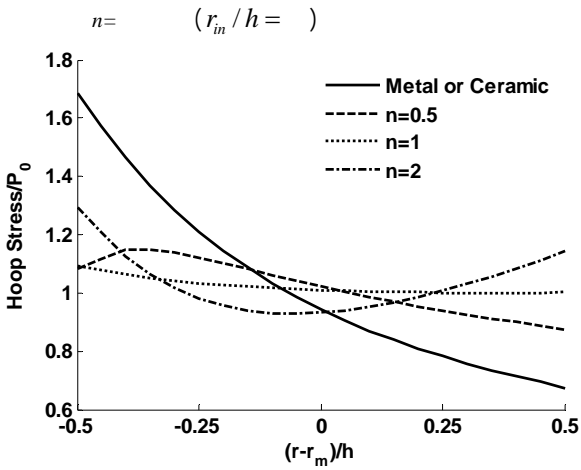
$\times$  / °C

[ ]





GPa  
 / W/mK  
 × / °C



$n =$  ( $r_{in}/h =$  )

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:( )

$P_0 =$  MPa

FGM ( ) ( )  $n$

$n =$  ( ) ( $r_{in}/h =$  )

FGM

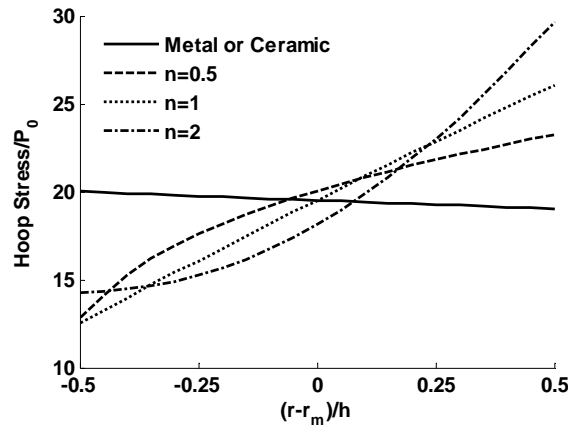
FGM

( ) ( )  $n$

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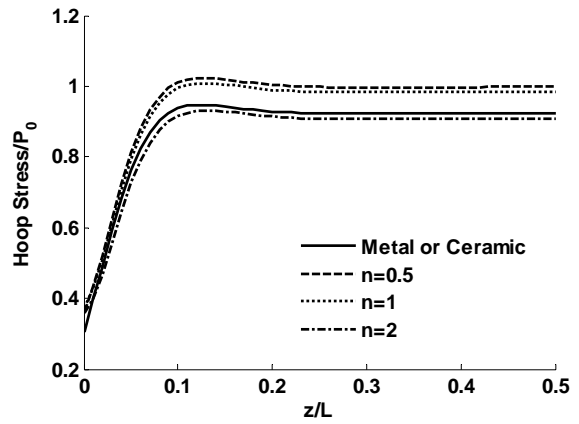


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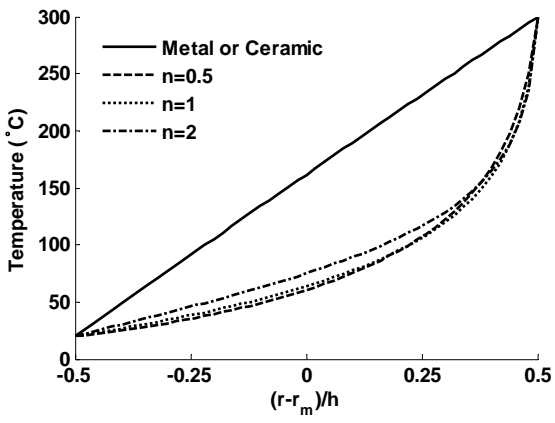
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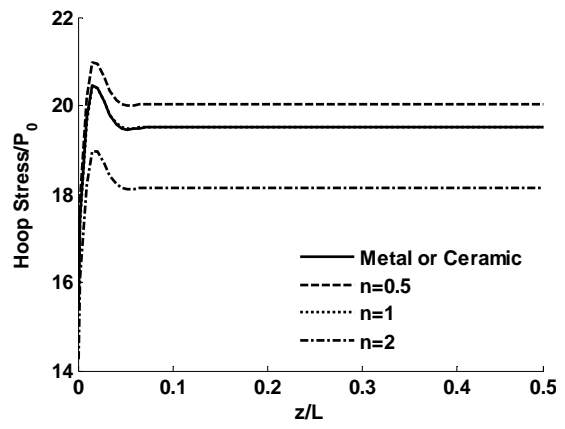


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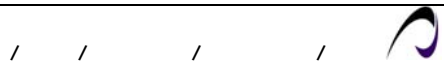
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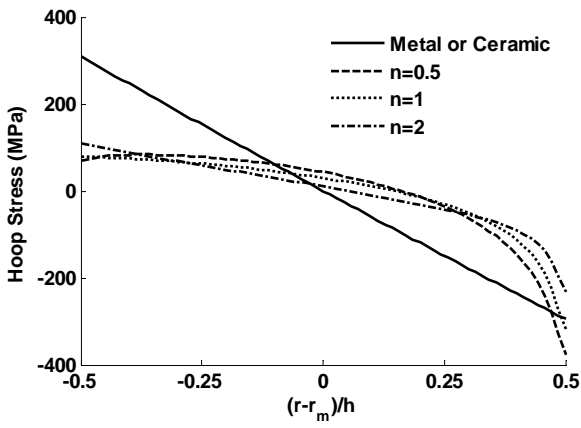


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FGM



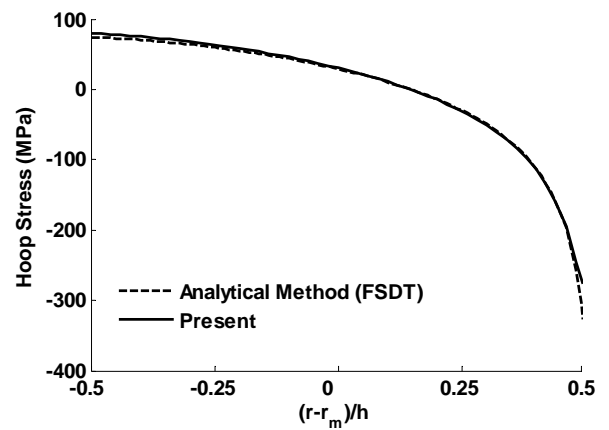


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( )  $n=1$

$n$

( ) ( )



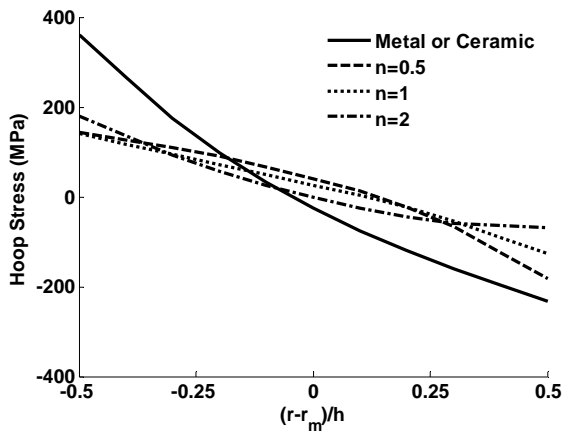
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$n=$

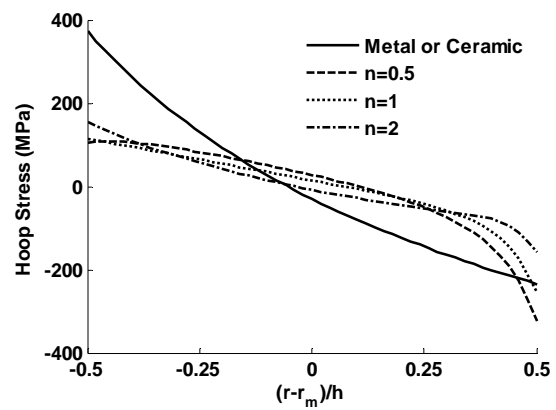
$P_0 =$  MPa

( )  $n$

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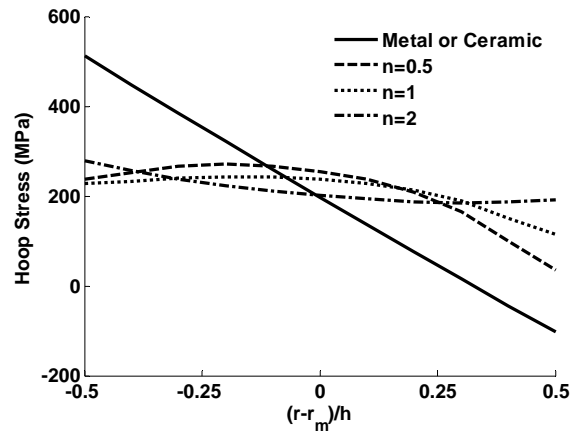
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