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## ***Modeling and Solving of a Bi-Criteria Scheduling Problem for a Cellular Manufacturing System with Sequence-Dependent Cell Setup Times***

R. Tavakkoli-Moghaddam; A. Nasri

### ***ABSTRACT***

This paper presents a new, bi-criteria mixed-integer programming model for scheduling cells (part families) and parts within each cell in a manufacturing cellular system. The objective of this model is to minimize the makespan and inter-cell movements simultaneously, while considering sequence-dependent cell setup times. In the CMS design and planning, three main steps must be considered, namely cell formation (i.e., part families and machine grouping), inter and intra-cell layouts, and scheduling issue. Due to the NP-hardness of the proposed model and the scheduling problem in the CMS, a genetic algorithm (GA) as an efficient meta-heuristic method is proposed to solve such a hard problem. Finally, a number of test problems are solved to show the efficiency of the proposed GA and the related computational results are compared with the results obtained by the use of an optimization tool.

**KEYWORDS :** Scheduling, Cellular Manufacturing System, Sequence-Dependent Setup Times, Genetic Algorithm

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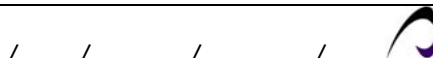
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زیر نویسها

$i = \{1, \dots, P\}$

: i

P

$j = \{1, \dots, M\}$

: j

M

$$\sum_{c \in C} y_{cb} = 1 \quad ; \forall b \in Kc \quad (3)$$

$$\sum_{b \in Kc} y_{cb} = 1 \quad ; \forall c \in C \quad (4)$$

$$\sum_{k \in K} z_{ikc} = x_{ic} \quad ; \forall i \in P, \forall c \in C \quad (5)$$

$$\sum_{i \in P} z_{ikc} \leq 1 \quad ; \forall k \in K, \forall c \in C \quad (6)$$

$$C_{11c1} = \max(\forall i \in P : (t_{i1} + s_{cc1} \times a_{i1}) \times z_{i1c} \times y_{c1}) \quad ; \forall c \in C \quad (7)$$

$$C_{k1c1} = \begin{cases} C_{k-1,1,c,1} + \max(\forall i \in P : t_{i1} \times z_{ikc} \times y_{c1}) & \text{if } C_{k-1,1,c,1} > 0 \\ \max(\forall i \in P : (t_{i1} + s_{cc1} \times a_{i1}) \times z_{ikc} \times y_{c1}) & \text{if } C_{k-1,1,c,1} = 0 \end{cases} \quad ; \forall (k \geq 2) \in K, c \in C \quad (8)$$

$$\min \sum_{i=1}^P \sum_{c=1}^C \left( \sum_{j=1}^M a_{ij} \times |a_{ij} - m_{jc}| \right) \times x_{ic} \times T_c + C_{\max} \quad (9)$$

$$\sum_{c \in C} x_{ic} = 1 \quad ; \forall i \in P \quad (10)$$

$$C_{1jc1} = C_{1,j-1,c,1} + \max(\forall i \in P : (t_{ij} + s_{cej} \times a_{ij}) \times z_{ilc} \times y_{c1}) ; \forall (j \geq 2) \in M \ \& \ \forall c \in C \quad (11)$$

(j-1)  
(b-1)

$$j \quad (b) \quad j$$

$$C_{1jcb} = \max(\max(\forall n \in C : C_{K,j,n,b-1}), C_{1,j-1,c,b}) + \max(\forall n \in C, n \neq c \ \& \ i \in P : (t_{ij} + s_{nej} \times a_{ij} \times y_{n,b-1}) \times z_{ilc} \times y_{cb}) ; \forall (j \geq 2) \in M, \ c \in C, \ (b \geq 2) \in Kc \quad (12)$$

(d<k)

$$j \quad ( )$$

$$j \quad (k-1) \quad (k)$$

(k-1)

$$(j-1) \quad (k) \quad j$$

$$C_{kjc1} = \begin{cases} \max(C_{k-1,j,c,1}, C_{k,j-1,c,1}) + \max(\forall i \in P : t_{ij} \times z_{ike} \times y_{c1}) & \text{if } C_{k-1,j,c,1} > 0 \\ \max(C_{k-1,j,c,1}, C_{k,j-1,c,1}) + \max(\forall i \in P : (t_{ij} + s_{cej} \times a_{ij}) \times z_{ike} \times y_{c1}) & \text{if } C_{k-1,j,c,1} = 0 \end{cases} ; \forall (k \geq 2) \in K, \ (j \geq 2) \in M, \ c \in C \quad (13)$$

$$( ) \quad j \quad ( ) \quad k$$

$$j \quad (k-1) \quad (k)$$

$$j \quad (j-1) \quad (k)$$

(d<k)

j

$$C_{1,1,C,b} = \begin{cases} \max(\forall n_1 \in C, n_1 \neq c \ \& \ i \in P : C_{K,1,n_1,b-1} + (s_{n_1c1} \times a_{i1} \times y_{n_1,b-1} + t_{i1}) \times z_{i1c} \times y_{cb}) & \text{if } C_{K,1,n_1,b-1} > C_{K,1,n_2,b-2} > 0 \\ \max(\forall n_1 \in C, n_1 \neq c \ \& \ i \in P : C_{K,1,n_1,b-1} + (s_{n_2c1} \times a_{i1} \times y_{n_2,b-1} + t_{i1}) \times z_{i1c} \times y_{cb}) & \text{if } C_{K,1,n_1,b-1} = C_{K,1,n_2,b-2} > C_{K,1,n_3,b-3} > 0 \\ \vdots \\ \max(\forall n_1 \in C, n_1 \neq c \ \& \ i \in P : C_{K,1,n_1,b-1} + (s_{n_f c1} \times a_{i1} \times y_{n_f,b-f} + t_{i1}) \times z_{i1c} \times y_{cb}) & \text{if } C_{K,1,n_1,b-1} = C_{K,1,n_2,b-2} = \dots = C_{K,1,n_f,b-f} > 0 \\ \max(i \in P : (s_{cc1} \times a_{i1} + t_{i1}) \times z_{i1c} \times y_{cb}) & \text{if } C_{K,1,n_1,b-1} = 0 \end{cases} ; \forall (b \geq 2) \in Kc \ \& \ c \in C \quad (9)$$

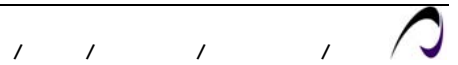
$$C_{k1cb} = \begin{cases} C_{k-1,1,c,b} + \max(\forall n \in C, n \neq c \ \& \ i, o \in P, i \neq 0 \ \& \ d \in K, d < k : (s_{ncl} \times a_{il} \times y_{n,b-1} \times (1 - a_{o1} \times z_{odc}) + t_{i1}) \times z_{ike} \times y_{cb}) & \text{if } C_{k-1,1,c,b} > 0 \\ \max(i \in P : (s_{cc1} \times a_{i1} + t_{i1}) \times z_{ike} \times y_{cb}) & \text{if } C_{k-1,1,c,b} = 0 \end{cases} ; \forall (b \geq 2) \in Kc, \ c \in C, \ (k \geq 2) \in K \quad (10)$$

$$( ) \quad j \quad ( )$$

$$j \quad (j-1)$$

k

j



$$C_{k_j c b} = \max(C_{k-1, j, c, b}, C_{k, j-1, c, b}) + \max(\forall n \in C, n \neq c \quad (14)$$

$$\& i, o \in P, i \neq 0 \& d \in K, d \leq k: (s_{ncj} \times a_{ij} \times$$

$$y_{n, b-1} \times (1 - a_{oj} \times z_{ode}) + t_{ij}) \times z_{ikc} \times y_{cb}$$

$$; \forall (j \geq 2) \in M, c \in C, (k \geq 2) \in K, (b \geq 2) \in K_c$$

$$C_{max} ( )$$

نمایش جواب

(c × p) A

$$C_{max} = \max(C_{k, j, c, b}) \quad (15)$$

$$; \forall j \in M, k \in K, c \in C \& b \in K_c$$

$$( )$$

p

p

$$x_{ic}, y_{cb}, z_{ikc} : \text{binary} ; \forall i \in P, k \in K, c \in C, b \in K_c \quad (16)$$

A  
( × c) B

B A

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B

B A

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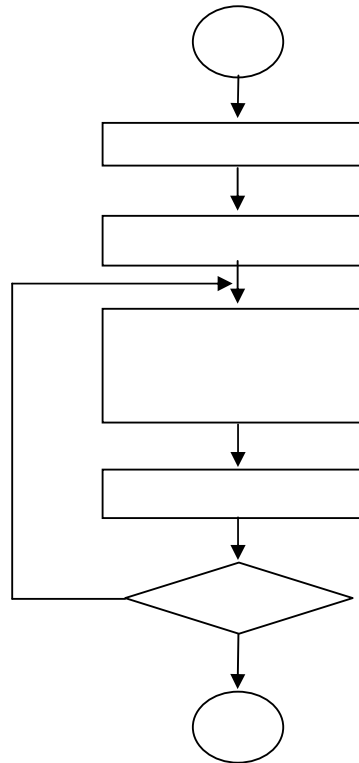
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GA

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جمعیت اولیه



$P_c$

$$pop-size = \begin{cases} 2 \times (m \times p \times c) & \text{if } (m \times p \times c) \leq 40 \\ 80 & \text{if } (m \times p \times c) > 40 \end{cases} \quad (18)$$

تابع برازش

$$g$$

( )

$$f_i = \begin{cases} C_{max} - g_i & \text{if } g_i < C_{max} \\ 0 & \text{otherwise} \end{cases} ; \quad i = 1, \dots, n \quad (19)$$

$$C_{max} \quad n = pop-size$$

$g_i$

A

$P_2 \quad P_1$

$C_2 \quad C_1$

$C_{max} \quad [ \quad ] \quad k$

$g$

مکانیزم نمونه گیری

$$\begin{array}{l}
 P_1 \quad \begin{array}{cccccc} 0 & 2 & 5 & 0 & 0 & \\ 3 & 0 & 1 & 0 & 4 & \end{array} \rightarrow C_1 \quad \begin{array}{cccccc} 0 & 2 & 5 & 2 & 1 & \\ 3 & 0 & 1 & 0 & 4 & \end{array} \\
 P_2 \quad \begin{array}{cccccc} 5 & 0 & 0 & 2 & 1 & \\ 0 & 0 & 3 & 0 & 4 & \end{array} \rightarrow C_2 \quad \begin{array}{cccccc} 5 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 3 & 0 & 4 & \end{array}
 \end{array}$$

$f_k$

$k$

$$C_1 \quad ( \quad )$$

$$C_2 \quad ( \quad )$$

$$p_k = f_k / \sum_{i=1}^n f_i \quad (n = pop-size) \quad (20)$$

SSR

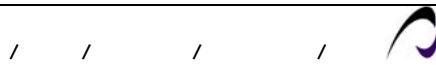
عملگر جهش

$(P_m)$

عملگر تقاطع

$P_m$

$(P_c)$



$$\begin{array}{cc}
 C_1 & \begin{array}{ccccc} 0 & \underline{0} & 5 & 2 & 1 \\ 3 & 0 & \underline{0} & 0 & 4 \end{array} & I_1 & \begin{array}{ccccc} 0 & \underline{2} & 0 & 0 & 0 \\ 0 & 0 & \underline{1} & 0 & 0 \end{array} \\
 \\
 C_2 & \begin{array}{ccccc} 5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 4 \end{array} & I_2 & \begin{array}{ccccc} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \\
 \\
 C_2 & & I_1 & & & \\
 \\
 C'_2 & C'_1 & & C_1 & & I_2 \\
 \\
 C'_1 & \begin{array}{ccccc} 0 & 0 & 5 & 2 & 1 \\ 3 & 0 & 0 & 0 & 4 \end{array} & C'_2 & \begin{array}{ccccc} 5 & 0 & 2 & 0 & 0 \\ 1 & 0 & 3 & 0 & 4 \end{array} & & C_1
 \end{array}$$

معیار توقف

$$\begin{array}{ccc}
 & & C'_1 \\
 & \downarrow & \downarrow \\
 C_1 & \begin{array}{ccccc} 0 & 2 & 5 & 2 & 1 \\ 3 & 0 & 1 & 0 & 4 \end{array} & \rightarrow & C'_1 & \begin{array}{ccccc} 0 & 2 & 2 & 5 & 1 \\ 3 & 0 & 0 & 1 & 4 \end{array}
 \end{array}$$

طراحی آزمایش

( )

$I_2$   $I_1$   $C_2$   $C_1$

U[1,10]





:( )

$(M \times P \times C)$	
800	
1800	
3600	

( )

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$(a_{ij})$  : ( )

5	4	3	2	1	
.	.	1	1	1	1
.	.	.	1	1	2
1	1	1	.	.	3
.	1	1	.	.	4

$(t_{ij})$  : ( )

5	4	3	2	1	
.	.	12	5	4	1
.	.	.	4	6	2
14	6	7	.	.	3
.	3	5	.	.	4

$(m_{cj})$  : ( )

5	4	3	2	1	
.	.	.	1	1	1
1	1	1	.	.	2

$(s_{ncj})$  : ( )

5	4	3	2	1	
17	3	8	7	10	1-1
6	10	9	14	5	2-1
10	10	10	3	8	1-2
9	5	9	18	12	2-2

U[1,20]

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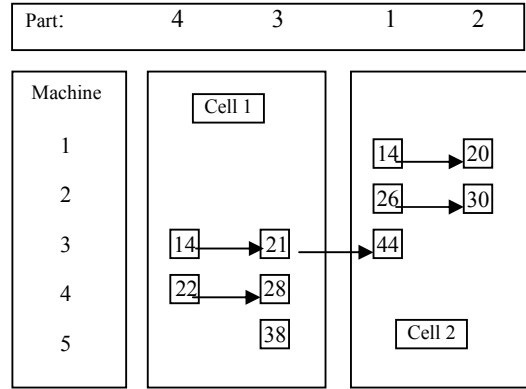
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$$(C_{max} = )$$

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تحليل نتایج

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2.4 GHz

512 MB RAM

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\* تا این زمان، جواب بهینه‌ای برای مسأله توسط نرم‌افزار لینگو ۸ به دست نیامده است.

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<sup>1</sup> Cellular Manufacturing Systems

<sup>2</sup> Non-Deterministic Polynomial Hard (NP-Hard)

<sup>3</sup> Genetic Algorithm

<sup>5</sup> Multi-Criteria Mixed-Integer Programming Model

<sup>5</sup> Crossover

<sup>6</sup> Mutation

<sup>∧</sup> Stochastic Sampling with Replacement (SSR)

<sup>9</sup> Roulette Wheel Selection

<sup>10</sup> Small Setup Time (SSU)

