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An Investigation of Condensing Steam Flow in a Turbine Cascade with Injection of Water Droplets at Inlet

A.R. Teymourtash, M.R. Mahpaker, E. Lakzian

ABSTRACT

During the course of expansion of steam in a Laval nozzle and a cascade of turbine, the state path crosses the saturation line; the steam first supercools and then reverts to equilibrium through the spontaneous formation of droplets or condensation shock, which causes aerodynamics and thermodynamics losses. In this way by formation of droplets and so reducing Gibbs energy, equilibrium is reached. This paper describes a two-phase model and provides an approach for including spontaneous homogeneous nucleation. In order to solve conservation equations, coupled with the equations of formation and growth of the droplets, a 2-D time-marching solution scheme with Baldwin-Lomax

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Teymourtash@um.ac.ir

mahpeymr@um.ac.ir

e.lakzian@sttu.ac.ir

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turbulence model, was used in this study. Pressure distribution and droplets size in a Laval nozzle and a turbine cascade are predicted and compared with empirical results. In the strength of validation, the effect of injection of water droplets into the steam flow in order to control the intensity and location of condensation shock is considered theoretically. A converging nozzle is used to producing droplets at inlet of turbine cascade. The results illustrate that wet steam at inlet of a turbine cascade weakens or delays the condensation shock in the passage of the blades.

KEYWORDS : Supercooled, Nucleation, Condensation Shock, Wet Steam, Jameson, Baldwin-Lomax, Injection of Droplets.

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CFX-TASC flow

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(A-B)

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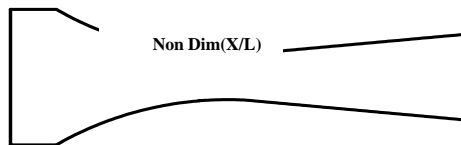
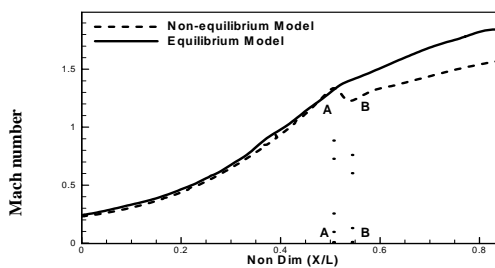
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$$\Delta G = 0$$

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$$r^* = \frac{2\sigma_r}{\rho_L RT_G \ln\left(\frac{P}{P_s(T_G)}\right)} \quad (1)$$

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$$J_{st} = J_{class} = q_c \frac{\rho_G^2}{\rho_L} \sqrt{\left(\frac{2\sigma_r}{\pi M^3}\right)} \exp\left[-\frac{\Delta G^*}{KT_G}\right] \quad (2)$$

q_c

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$$n_g = n_1 \exp\left(\frac{-\Delta G}{KT_G}\right) \quad (3)$$

n_g
K (Clusters/m³)

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$$J_{Ka} = \frac{1}{1+\phi} J_{st} \quad (4)$$

$$\phi = \frac{q_c \rho_G}{\alpha_r} \left(\frac{RT_G}{2\pi}\right)^{0.5} \left(\frac{L^2}{RT_G^2} - \frac{L}{2T_G}\right) \quad (5)$$

α_r

$$L = h_G - h_L \quad (6)$$

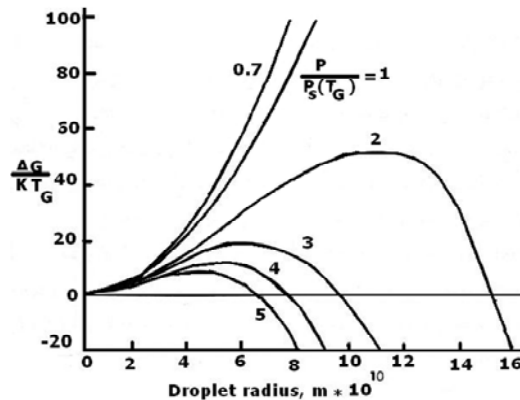
[] α_r

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$$\alpha_r = \frac{\lambda}{r(1+3.18K_n)} \quad (7)$$

λ

[]



(:)

[] (P/P_s(T_G))

100°C

100°C

$$\frac{\partial W}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} = \frac{\partial R}{\partial x} + \frac{\partial S}{\partial y}$$

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$$G = \begin{bmatrix} \rho v \\ \rho v u \\ P + \rho v^2 \\ \rho v h_0 \end{bmatrix} \quad F = \begin{bmatrix} \rho u \\ P + \rho u^2 \\ \rho u v \\ \rho u h_0 \end{bmatrix} \quad W = \begin{bmatrix} \rho \\ \rho v \\ \rho u \\ \rho e_0 \end{bmatrix} \quad ()$$

$$R = \begin{bmatrix} 0 \\ \sigma_x \\ \tau_{yx} \\ \sigma_x u + \tau_{xy} v + k \frac{\partial T}{\partial x} \end{bmatrix} \quad S = \begin{bmatrix} 0 \\ \tau_{xy} \\ \sigma_y \\ \sigma_y v + \tau_{xy} u + k \frac{\partial T}{\partial y} \end{bmatrix}$$

G F

S R

W

$$K_n = \frac{1.88 \mu_G}{2r \rho_G \sqrt{RT_G}} \quad ()$$

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$$L \frac{dm_r}{dt} = 4\pi r^2 \alpha_r (T_L - T_G) \quad ()$$

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k, h_0, e_0

$$T_L = T_G + \left[1 - \frac{r^*}{r} \right] [T_s(p) - T_G] \quad ()$$

$\sigma_x, \sigma_y, \tau_{xy}, \tau_{yx}$

$T_G \quad T_L$

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$$\sigma_x = -\frac{2}{3} \mu_{eff} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + 2\mu_{eff} \frac{\partial u}{\partial x} \quad ()$$

$$\sigma_y = -\frac{2}{3} \mu_{eff} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + 2\mu_{eff} \frac{\partial v}{\partial y} \quad ()$$

$$\tau_{xy} = \tau_{yx} = \mu_{eff} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \quad ()$$

μ_{eff}

$$\frac{a}{2} (r^2 - r_1^2) + b(r - r_1) + c \ln \left(\frac{r - r^*}{r_1 - r^*} \right) = d \delta t \quad ()$$

t_1

r_1

r^*

$\delta t = t - t_1$

$$\mu_{eff} = \mu + \mu_t \quad ()$$

$$a = h_G - c_L (T_s(P) - T_D)$$

$$b = (r^* + 1.59\bar{l}) \times$$

$$[h_G - c_L (T_s(P) - T_D) + c_L r^* (T_s(P) - T_G)]$$

$$c = r^* (r^* + 1.59\bar{l}) [h_G - c_L (T_G - T_D)]$$

$$d = \frac{\lambda}{T_s(P) - T_G}$$

$$T_s(P)$$

c_L

h_G

T_D

$\bar{l} \quad 273.16 K$

T_G

$$\frac{\mu(\rho, T)}{\mu^*} = \psi(\delta, \theta) = \psi_0(\theta) \psi_1(\delta, \theta) \quad ()$$

$$\delta = \frac{\rho}{\rho^*}, \quad \theta = \frac{T}{T^*}, \quad \mu^* = 1 \times 10^{-6} Pa s \quad ()$$

$\psi_0, \psi_1 \quad \psi$

ψ_0



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$$\psi_1(\delta, \theta) = \exp \left[\delta \sum_{i=1}^{21} n_i (\delta - 1)^{i-1} (\theta - 1)^{i-1} \right] \quad (1)$$

$$\psi_0(\theta) = \theta^{0.5} \left[\sum_{i=1}^4 n_i^0 \theta^{1-i} \right]^{-1} \quad (2)$$

$$\delta = \frac{\rho}{\rho^*}, \rho^* = \rho_c = 322 \text{ kg m}^{-3} \quad (3)$$

$$\theta = \frac{T}{T^*}, T^* = T_c = 647.096 \text{ K} \quad (4)$$

$$\theta = \frac{T}{T^*}, T^* = T_c = 647.096 \text{ K} \quad (5)$$

n_i^0

ψ_1

$[\] \psi_0$

$:()$

i	n_i^0	i	n_i^0
/	* ()	/	* ()
/	* ()	/	* ()

$[\] \psi_1$

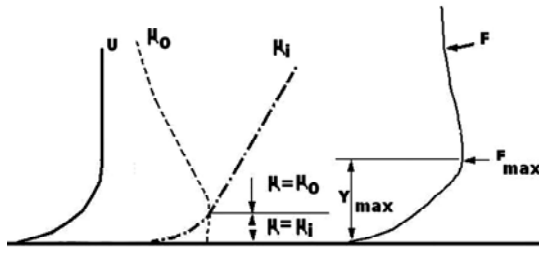
$:()$

i	I_i	J_i	n_i	i	I_i	J_i	n_i
/			/	/			/
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$$\mu_i = \rho (KYD)^2 |\omega| \quad (6)$$

$$Y = \frac{K = 0.4}{D} \rho \quad (7)$$

$$D = 1 - \exp \left(\frac{-Y^+}{26} \right) \quad (8)$$



$$\mu_i = \min(\mu_i, \mu_o) \quad (9)$$

μ_o μ_i



$$() \quad Y^+ = \frac{Y}{\mu_w} \sqrt{\rho_w |\tau_w|} \quad ()$$

$$\omega \quad \mu_w \quad \tau_w$$

$$\mu_0 = \rho k C_{cp} F_{wake} F_{kleb} \quad ()$$

$$k = 0.0168, C_{cp} = 1.6$$

$$F_{kleb} = \left(1 + 5.5 \left(C_{kleb} \frac{Y}{Y_{max}} \right)^6 \right)^{-1} \quad ()$$

$$C_{kleb} = 0.3$$

$$F_{wake} = \min \left(Y_{max} F_{max}, C_{wk} Y_{max} \frac{U_{diff}^2}{F_{max}} \right) \quad ()$$

$$F \quad F_{max} \quad C_{wk} = 0.25$$

()

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$$h = w h_L + (1-w) h_G \quad ()$$

$$\frac{1}{\rho} = \frac{w}{\rho_L} + \frac{(1-w)}{\rho_G} \quad ()$$

$$F(Y) = Y |\omega| D \quad ()$$

$$F(Y) \quad Y_{max}$$

$$U_{diff} = \left(\sqrt{u^2 + v^2} \right)_{max} - \left(\sqrt{u^2 + v^2} \right)_{min} \quad ()$$

$$v_{min} \quad v_{max} \quad u$$

$$v_L = \frac{v_C + a(T_C - T_S)^3 + b(T_C - T_S) + c(T_C - T_S)^4}{1 + d(T_C - T_S)^3 + e(T_C - T_S)} \quad ()$$

$$647.27^\circ K \quad (T_C) \quad 3.1975 \text{ cm}^3/\text{g}$$

$$w = \frac{M_L}{M_L + M_G} \quad ()$$

$$L \quad G \quad ()$$

$$a = -0.3151548$$

$$b = -1.203374 \times 10^{(-5)}$$

$$c = 7.48908 \times 10^{(-13)}$$

$$d = 0.1342489$$

$$e = -3.946263 \times 10^{(-3)}$$

in SI units

$$() \quad (T_L)$$

$$h_L = c_L (T_L - T_D) \quad ()$$

$$273.15^\circ K$$

$$T_D$$



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$$e = e_0 - \frac{u^2 + v^2}{2} \quad (43) \quad \left(\frac{\partial h_G}{\partial P_G} \right) = v_G - T_G \left(\frac{\partial v_G}{\partial T_G} \right)_P \quad ()$$

$$(T_G) \quad h_G = \int \left[v_G - T_G \left(\frac{\partial v_G}{\partial T_G} \right) \right] dp + F_h(T_G) \quad ()$$

$$(T_G, P) \quad () \quad ()$$

$$() \quad h_G = \frac{RT_G}{2} \left(\sqrt{1 + \frac{4PB}{RT_G}} - 1 \right) \left(1 - \frac{T_G}{B} \frac{dB}{dT_G} \right) + F_h(T_G) \quad ()$$

$$() \quad T_G \quad F_h(T_G) = a \ln T_G + bT_G + cT_G^2 - dT_G^3 + eT_G^4 - fT_G^5 + const. \quad ()$$

$$F(T_G) = \left(e_0 - \frac{u^2 + v^2}{2} \right) - \left(h_G - \frac{P}{\rho_G} \right) \quad (44)$$

$$() \quad \begin{aligned} a &= 46 & b &= 1.47276 \\ c &= 0.419465 \times 10^{-3} & d &= 7.33297 \times 10^{-8} \\ e &= 6.16548 \times 10^{-11} & f &= 1.94063 \times 10^{-14} \\ & & const. &= 1782.24 \text{ in SI units} \end{aligned}$$

$$\Delta T_G = \frac{-F(T_G)}{dF(T_G)/dT_G} \quad (45) \quad []$$

$$T_{G(new)} = T_{G(old)} + \Delta T_G \quad (46)$$

$$|\Delta T_G / T_G| \leq \varepsilon$$

$$10^{-4} \quad \varepsilon$$

$$h_G \quad T_G \quad P \quad (\rho, \rho u, \rho v, \rho e_0) \quad ()$$

$$J_{min} = 10^{18} \text{ Nuclei/kg.sec}$$

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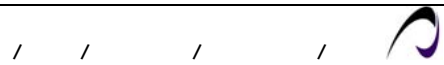
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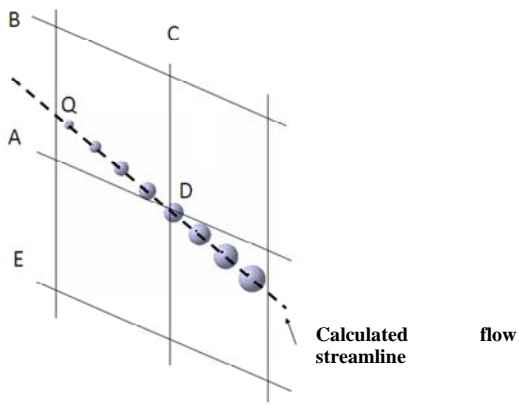
I. جریان خشک

$$() \quad D \quad \rho_G \quad (\rho) \quad h_G \quad (h) \quad ()$$

$$e = h_G - \frac{P}{\rho_G} \quad (47)$$

w





() :

$$F_1 = e_D - e'_D = \left[(1-w)h_G + wh_L - \frac{P}{\rho} \right] - e'_D$$

$$F_2 = \frac{1}{\rho_D} - \frac{1}{\rho'_D} = [(1-w)v_G + wv_L]_D - \frac{1}{\rho'_D} \quad (48)$$

$$F_3 = r_D - r'_D$$

D Q

D

جریان مرطوب .III

J_{min}

D)

r'_D, ρ'_D, e'_D

[] ()

F_1, F_2, F_3

$$r = -1.59\bar{I} + \sqrt{(1.59\bar{I})^2 + r_1(r_1 + 2(1.59\bar{I})) + 2\lambda \left(\frac{T_L - T_G}{h_G - h_L} \right) \delta t} \quad (49)$$

δt

r_1

$$\frac{\partial F_1}{\partial T_G}, \frac{\partial F_1}{\partial T_L}, \frac{\partial F_1}{\partial T_s(P)}$$

[]

QD

D

BAE

(B, A E)

Q

δt D

(47)

$$\delta t = \frac{\Delta X_D}{((u_Q + u_D)/2)}$$

Q

ΔX_D

D

N_Q

r_Q

Q

QD

(())

(())

T_L

T_G

$T_s(P)$

D

ρ_L, ρ_G, h_G
()

T_L

T_G

(



F_1, F_2

r'_D

()

()

[]

C

(

()

F_3

T'_{LD}

T_{LD}

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(

$$F_3 = T_{LD} - T'_{LD}$$

()

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C

(CFL)

()

C

(:)

$$\Delta t = FT \cdot CFL \cdot \frac{\Delta x}{(|V| + a)}$$

()

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|V|

Δx

CFL

FT

a

$$P_{0in} = 25 \text{ Kpa}$$

$$T_{0in} = 358.6 \text{ K}$$

$$P_{OUT} = 8 \text{ Kpa}$$

$$CFL = 2\sqrt{2}$$

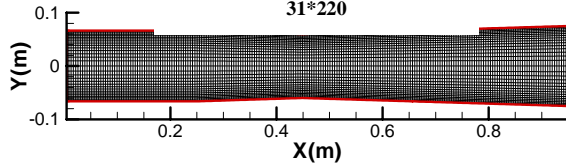
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Nozzle C- P0=25 kPa-T0=358.6 K

[19]

31*220

0.4 FT



C

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$$\left| \frac{\Delta u}{u} \right| < 0.01\%$$

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$$\left(\frac{P}{P_{0in}} \right)$$

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$$R_{error} = \frac{1}{N_{node}} \left[\sqrt{\sum_{ij} (\delta \rho_{ij}^2)} \right]$$

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$\delta \rho$

N_{node}

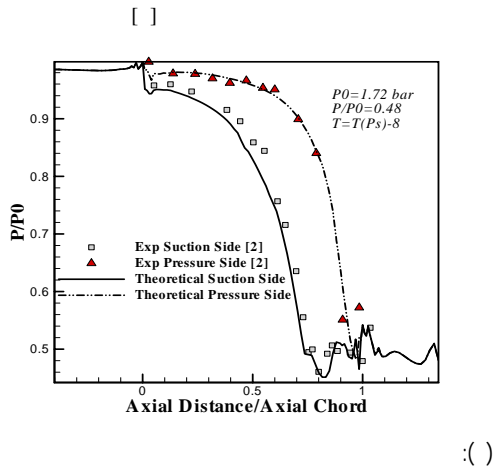
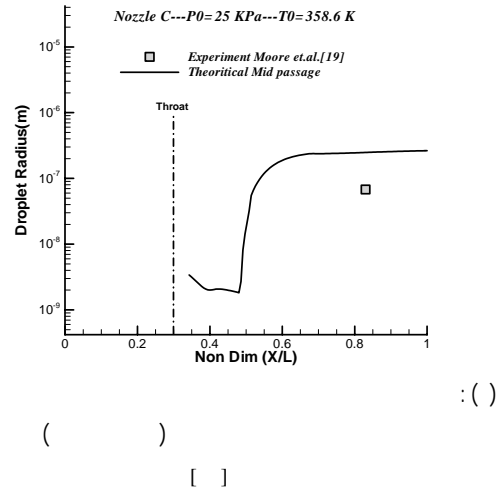
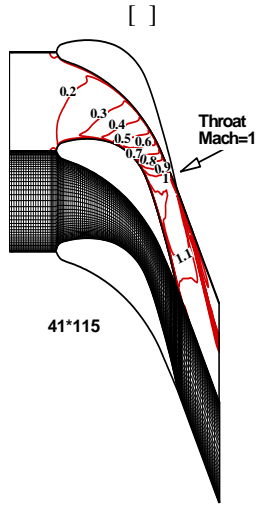
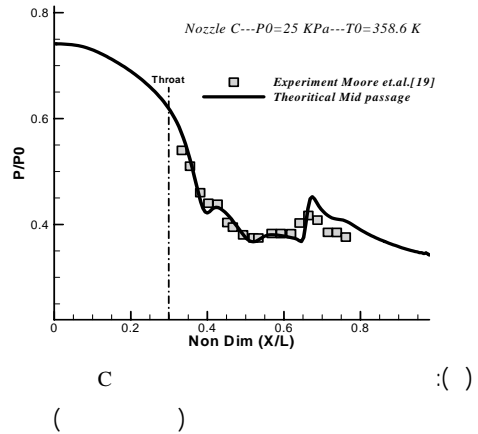
$$R_{error} \leq 1 \times 10^{-4}$$

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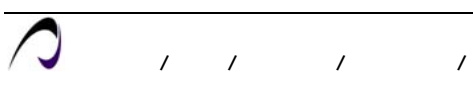
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$\left(\frac{P}{P_{0in}}\right)$
 $M = 1$
 $x/c = 0.7$
 ()
 []
 ()



$P_{0in} = 172 \text{ Kpa}$
 $T_{0in} = 380 \text{ K}$
 $P_{OUT} = 82.56 \text{ Kpa}$
 $\theta_{in} = 0^\circ$



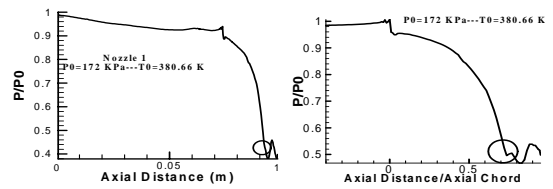
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n_g g

n_1 c

N e

M ΔG

\bar{l} h

P J

$P_S(T_G)$ T_G k

q_c K

(.) Kn)

R (

S_c D $S_c = \mu_G / \rho_G D$ L

(m_r



/ / / /

α_r		r	
μ_G		T	
μ_{eff}		$T_S(P)$	P
ρ		ΔT	$[T_s(P) - T_G]$
λ		t	
σ		u	x
$\rho_s(T_L, r)$	T_L	v	y
		w	
			z

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